Probabilistic downscaling to detect regional present and future climate hazards

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University of Warwick, ECMWF, University of Bristol

28th April 2020

Funding

- University of Warwick
- Alan Turing Institute

Introduction

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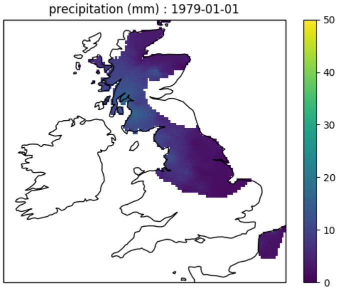

Figure: Precipitation

Introduction

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- Temperature
- Precipitable water content
- Humidity
- Geopotential
- Wind speed and velocity

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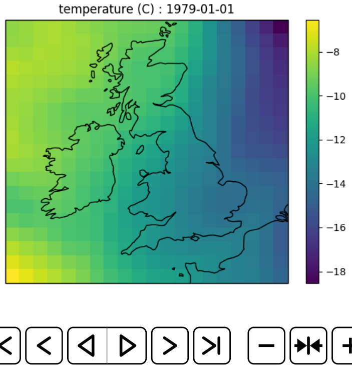
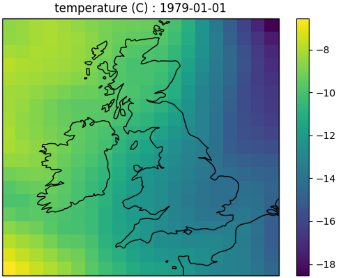
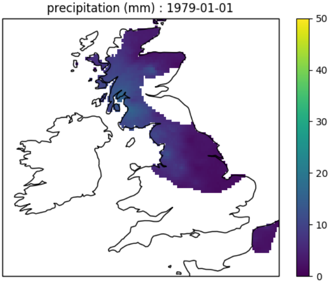


Figure: Air temperature

Introduction



(a) Model fields



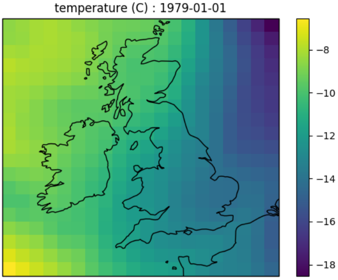
(b) Precipitation

Figure: Sample from data

Introduction

- 40 years of data (1979 - 2019)
- Observed precipitation ($\sim 10\,\text{km}$)
- Simulated model fields ($\sim 80\,\text{km}$)

Introduction



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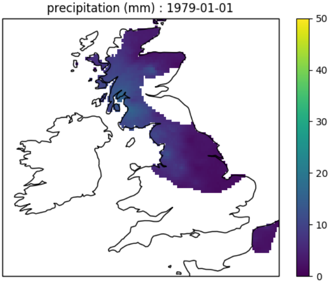

(b) Precipitation

Figure: Sample from data

Introduction

Can we use the model fields on a *coarse grid* to forecast the precipitation on the *fine grid*? This task is known as *downscaling*.

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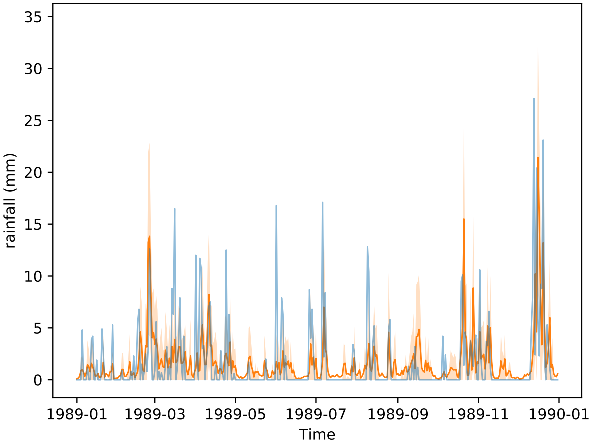

Figure: Forecast (orange) and observed (blue)

Introduction

$1.2 \pm 1.5\,\text{mm}$

Statistical model

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- Occurrence of precipitation

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- Quantity of precipitation
- Autocorrelation of precipitation
- Affected by the model fields

Statistical model

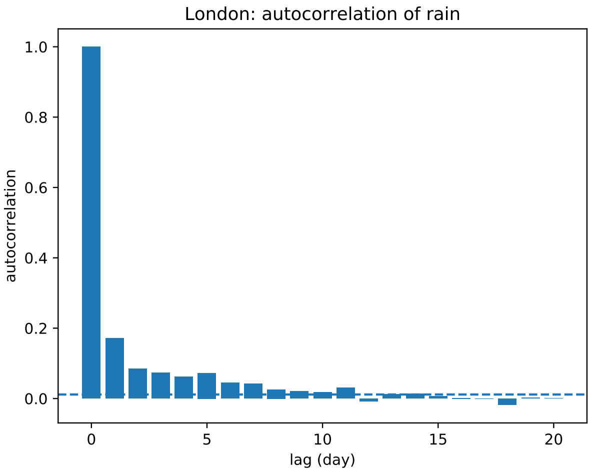

Figure: Autocorrelation of precipitation

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- We introduce the *compound-Poisson*.

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$$\omega_t = \exp\left[\boldsymbol{\beta}_\omega \mathbf{x_t} + k_\omega\right]$$

Statistical model

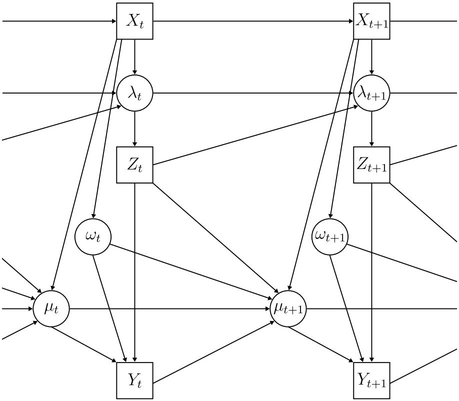
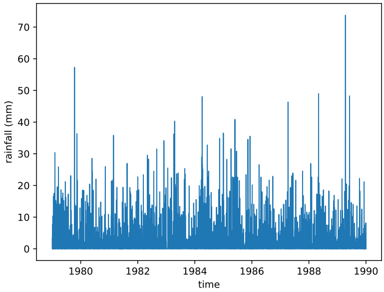
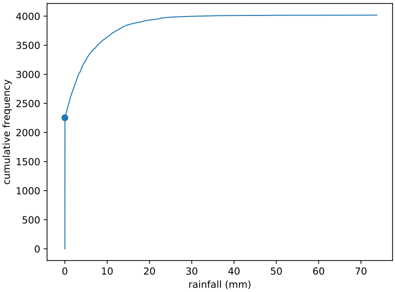


Figure: Graphical model

Statistical model



(a) Time series



(b) Cumulative

Figure: Simulated data

Statistical model

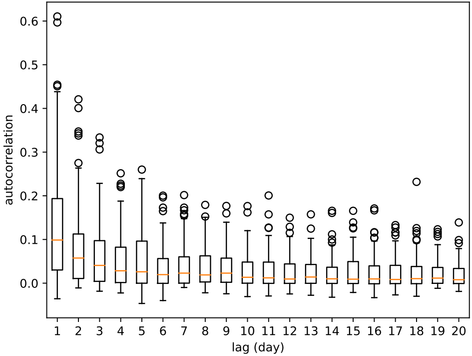


Figure: Simulated data autocorrelation ARMA(5,5)

Markov chain Monte Carlo

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- This was done using Markov chains Monte Carlo (MCMC).
- We use Monte Carlo which uses random numbers to solve a model fitting problem. Flucations in solutions, (samples), reflect the uncertainity.
- We use a Markov chain to draw dependent samples in such a way they converge to the right answer.

Markov chains Monte Carlo

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$$\text{posterior} \propto \text{likelihood} \times \text{prior}$$

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$$\underbrace{\pi(\beta, z_{1:T} | x_{1:T}, y_{1:T})}_{\text{posterior}} \propto \underbrace{p(y_{1:T}, z_{1:T} | x_{1:T}, \beta)}_{\text{likelihood}} \times \underbrace{\pi(\beta)}_{\text{prior}}$$

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$$\underbrace{p(\text{data}|\text{parameters})}_{\text{likelihood}}$$

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But the prior is subjective!

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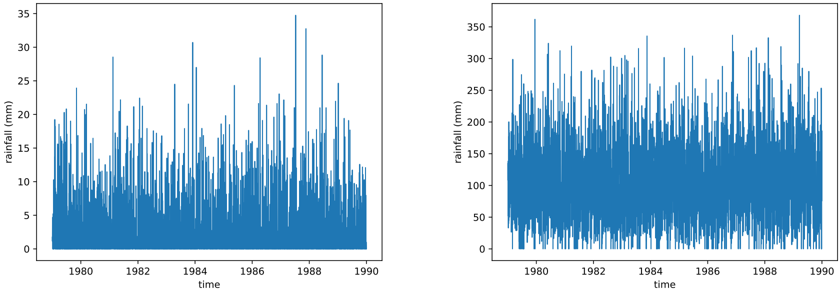

(a) Sensible (b) Not sensible

Figure: Simulated data

Markov chains Monte Carlo

We use Gibbs sampling to split the posterior into two parts

- Sample $Z_t | \beta, Z_{1:T\setminus t}, y_{1:T}, x_{1:T}$

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We use *slice sampling* and *elliptical slice sampling*.

Markov chains Monte Carlo

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Markov chains Monte Carlo

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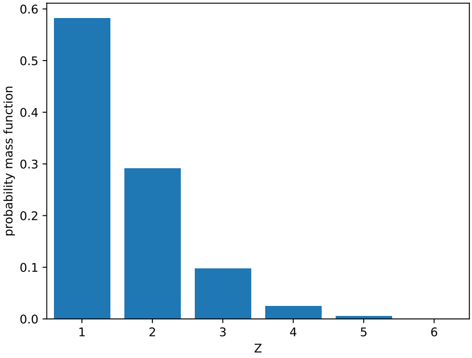
- $y_t = 0 \Rightarrow Z_t = 0$

Markov chains Monte Carlo

Slice sampling:

- $s \sim \text{Uniform}(0, \pi(z^{(i)}))$
- $z^{(i+1)} \sim \text{Uniform}(z : \pi(z) > s)$



Markov chains Monte Carlo

We use *elliptical* slice sampling to sample β.

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We use *elliptical* slice sampling to sample β. It can sample a posterior with a Normal prior. For the technical:

- Chain at $\beta^{(i)}$
- $\nu \sim$ prior
- $s \sim \text{Uniform}(0, \text{posterior}(\beta^{(i)}))$
- $\theta \sim \text{Uniform}(0, 2\pi)$
- $\beta^{(i+1)} = \beta^{(i)} \cos\theta + \nu \sin\theta$ if posterior$(\beta^{(i+1)}) > s$, else try another θ with a smaller support containing zero

Markov chains Monte Carlo

Even more technical:

- Initial: $[\theta_{\text{min}}, \theta_{\text{max}}] = [\theta - 2\pi, \theta]$
- If: $\theta < 0$, $\theta_{\text{min}} = \theta$
- Else: $\theta_{\text{max}} = \theta$

Markov chains Monte Carlo

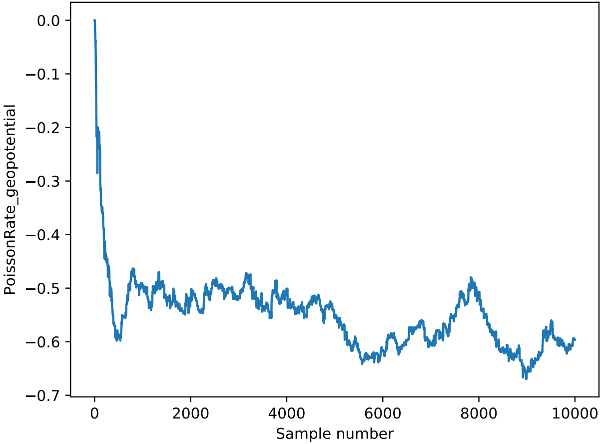

Figure: Example of a chain

Forecasting

Forecasting

Simulate the future using different samples from the posterior (MCMC chain). Any variation reflect the uncertainity.

Forecasting

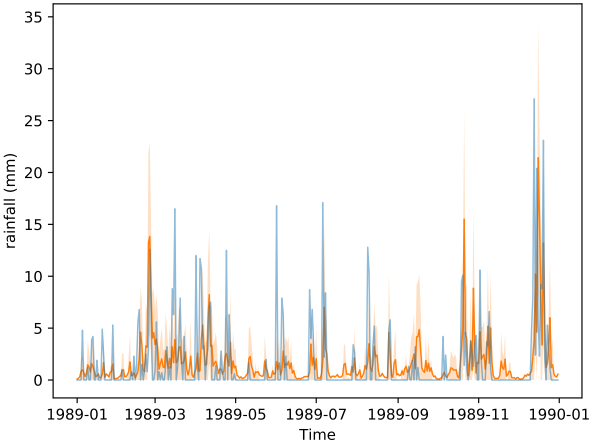

Figure: Forecast (orange) and observed (blue)

Forecasting

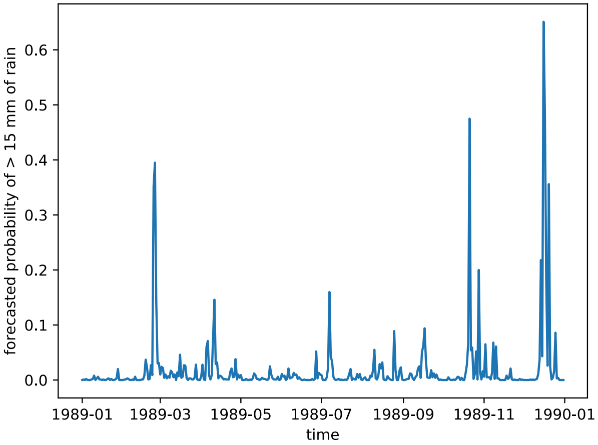

Figure: Probability precipitation $> 15\,\mathrm{mm}$

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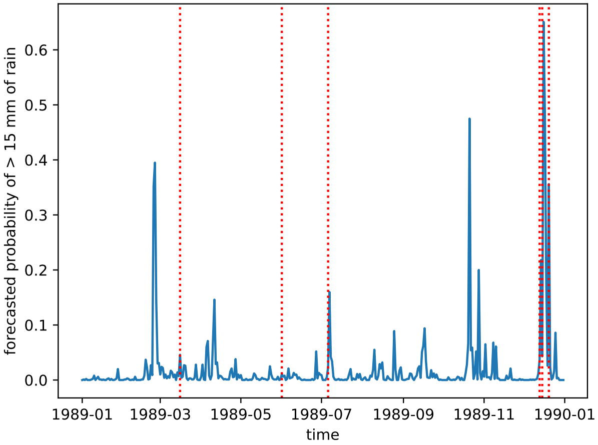

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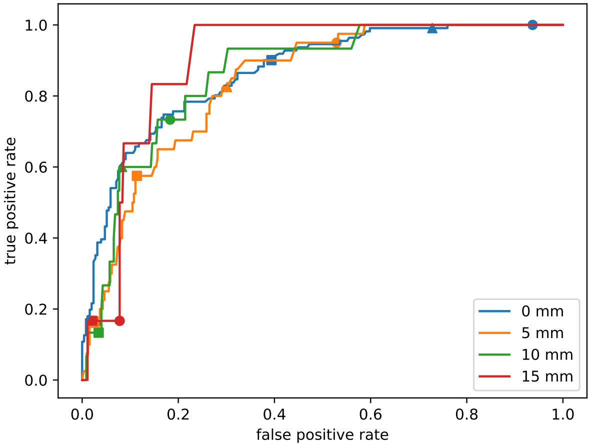

Figure: ROC curve

Downscaling

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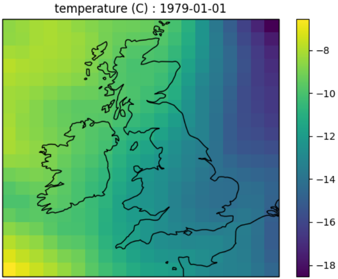
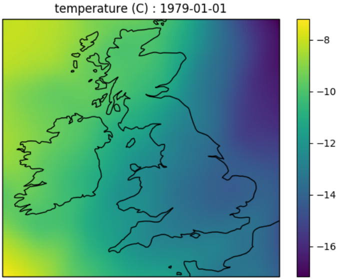
We interpolate the model fields for now



(a) Coarse grid



(b) Fine grid

Figure: Interpolation

Downscaling

We impose a prior on β where neighbouring locations with similar topography have similar values

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We impose a prior on β where neighbouring locations with similar topography have similar values

$$\begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_N \end{pmatrix} \sim \mathrm{N}\left(0, \tau^{-1}\mathbf{K}\right)$$

where $[\mathbf{K}]_{i,j} = \exp\left[-\frac{\nu}{2}\left(w_i - w_j\right)^2\right]$ and w_i is the topography

Downscaling

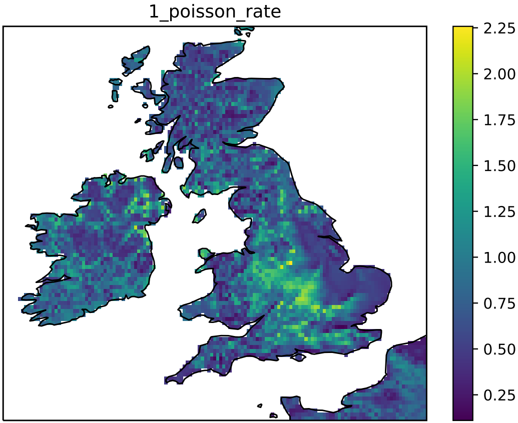

Figure: Simulation from prior

Downscaling

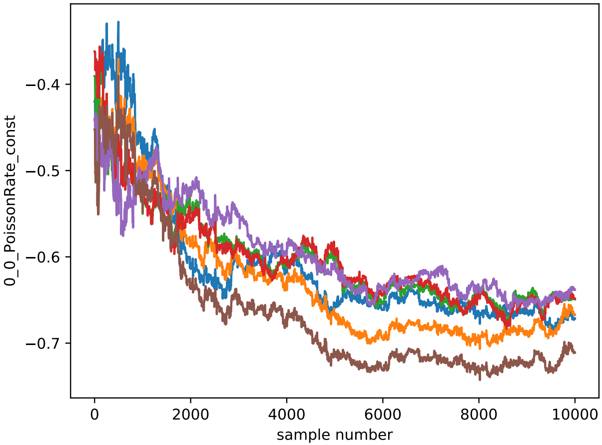

Figure: MCMC chains

Downscaling

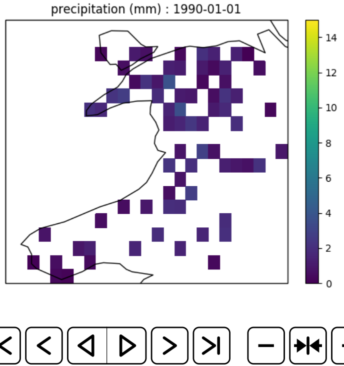

Figure: Forecast at Wales

Downscaling

Downscaling

Instead of interpolating the model fields, we can learn the model fields on the coarse grid. We use a *Gaussian process*.

Downscaling

- $x'_{i,t}$ = model fields at time t, location i on *coarse* grid
- $x_{j,t}$ = model fields at time t, location j on *fine* grid

$$\begin{pmatrix} x'_{1,t} \\ x'_{2,t} \\ \vdots \\ x'_{M,t} \end{pmatrix} \sim \mathrm{N}(0, \tau^{-1}\mathbf{K})$$

Downscaling

We can sample the model fields on the fine grid *given*:

- model fields on the coarse grid
- precipitation & Z on the fine grid

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Conclusion

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- We have quantified uncertainity.

Conclusion

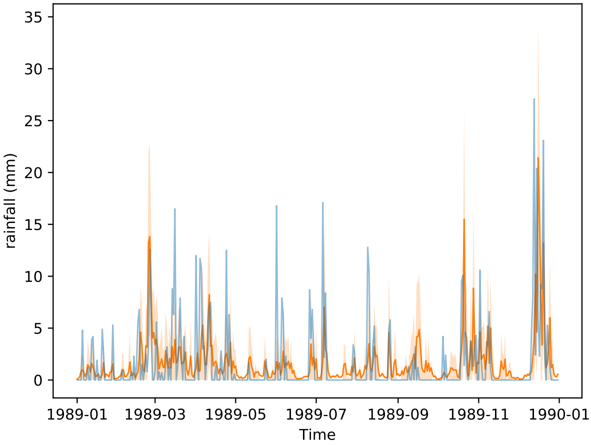

Figure: Forecast (orange) and observed (blue)

Conclusion

- The task of downscaling is computational expensive.

Conclusion

- To transistion from weather forecasting to climate prediction, we need to forecast many years into the future.

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Thank you