TC/PR/RB Lecture 1 – Introduction to ensemble prediction

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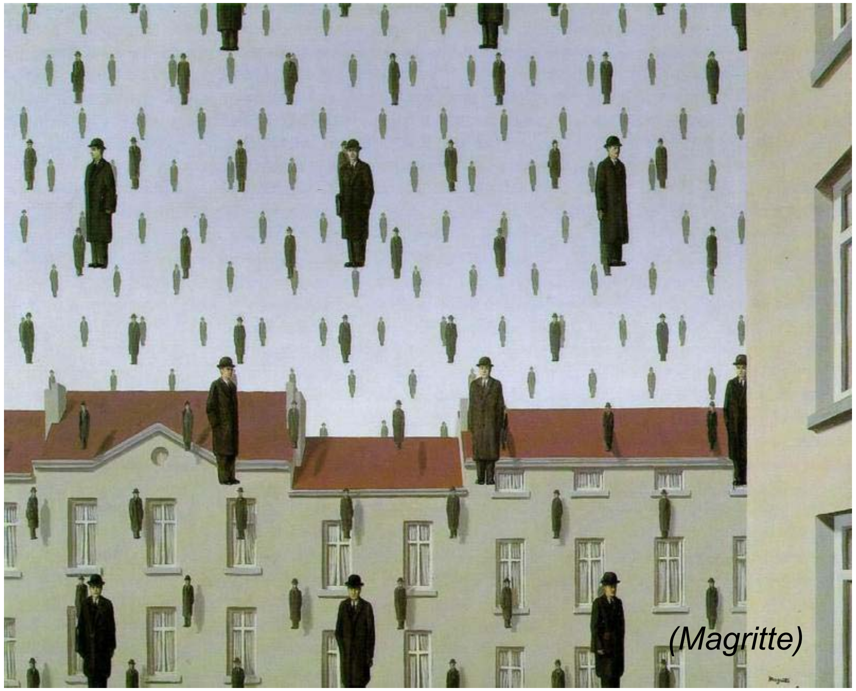

(Magritte)

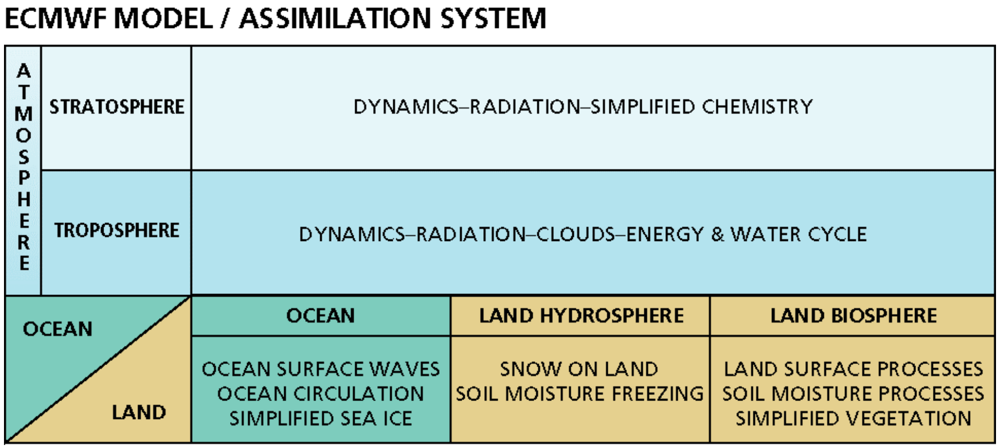
Outline

- Sources of forecast errors: initial and model uncertainties.
- Flow-dependent predictability.
- The probabilistic approach to NWP.
- Ensemble prediction as a practical tool to estimate the time evolution of the probability density function of forecast states.
- The simulation of initial uncertainties in ensemble prediction.
- Phase-space directions with maximum growth.
- Singular vectors and normal modes: introduction.

The ECMWF Numerical Weather Prediction (NWP) Model

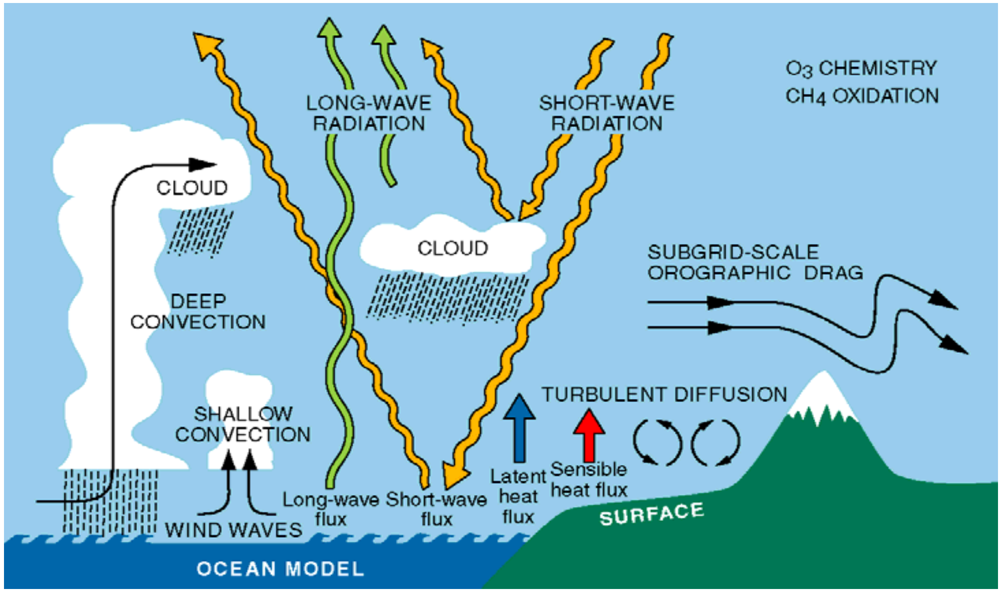
The behavior of the atmosphere is governed by a set of physical laws which express how the air moves, the process of heating and cooling, the role of moisture, and so on.

Interactions between the atmosphere and the underlying land and ocean are important in determining the weather.



ECMWF MODEL / ASSIMILATION SYSTEM

ATMOSPHERE	STRATOSPHERE	DYNAMICS–RADIATION–SIMPLIFIED CHEMISTRY		
	TROPOSPHERE	DYNAMICS–RADIATION–CLOUDS–ENERGY & WATER CYCLE		
OCEAN LAND		OCEAN	LAND HYDROSPHERE	LAND BIOSPHERE
		OCEAN SURFACE WAVES OCEAN CIRCULATION SIMPLIFIED SEA ICE	SNOW ON LAND SOIL MOISTURE FREEZING	LAND SURFACE PROCESSES SOIL MOISTURE PROCESSES SIMPLIFIED VEGETATION

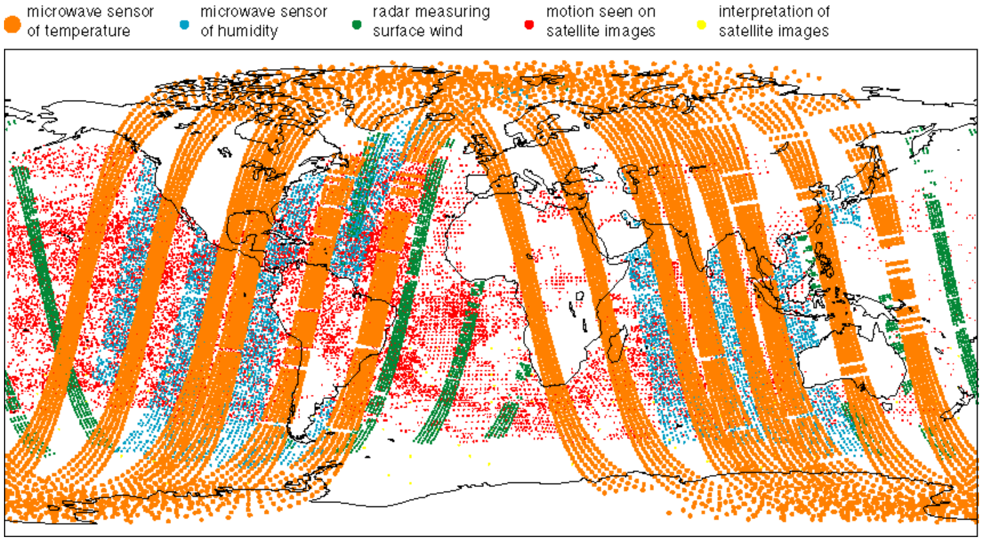


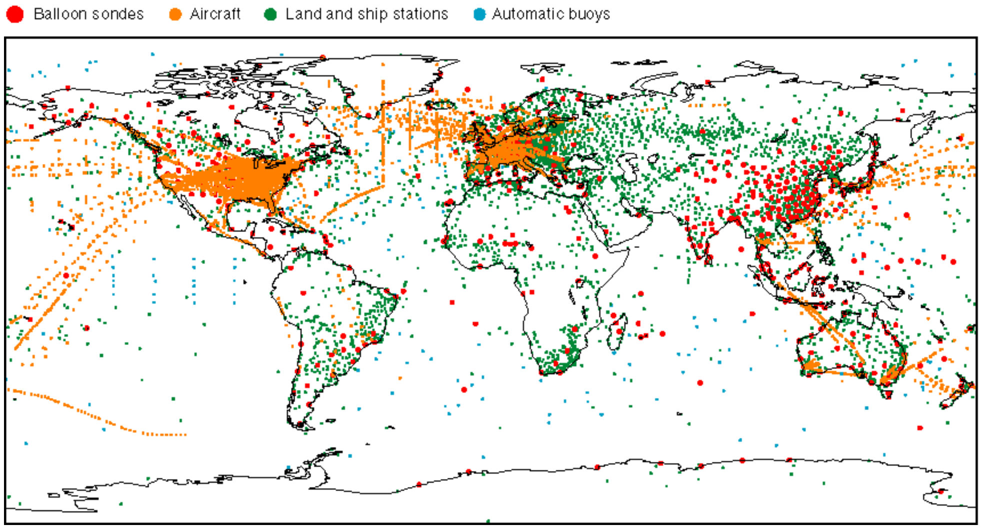
Starting a NWP: the initial conditions

To make accurate forecasts it is important to know the current weather:

- observations covering the whole globe are continuously downloaded and fed into the system;
- about 600,000 observations are processed every 12 hours;
- complex assimilation procedures are used to optimally define the initial state of the system.

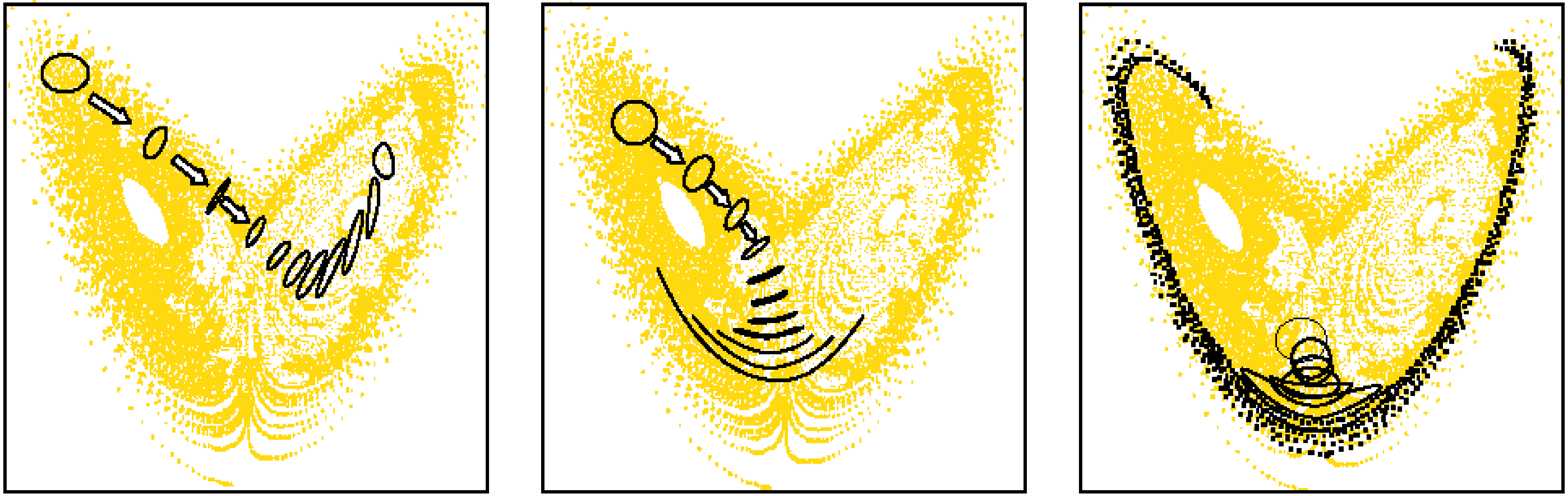
Unfortunately, very few observations are taken in some regions of the world (e.g. polar caps, oceans).





Sources of forecast errors: initial and model uncertainties

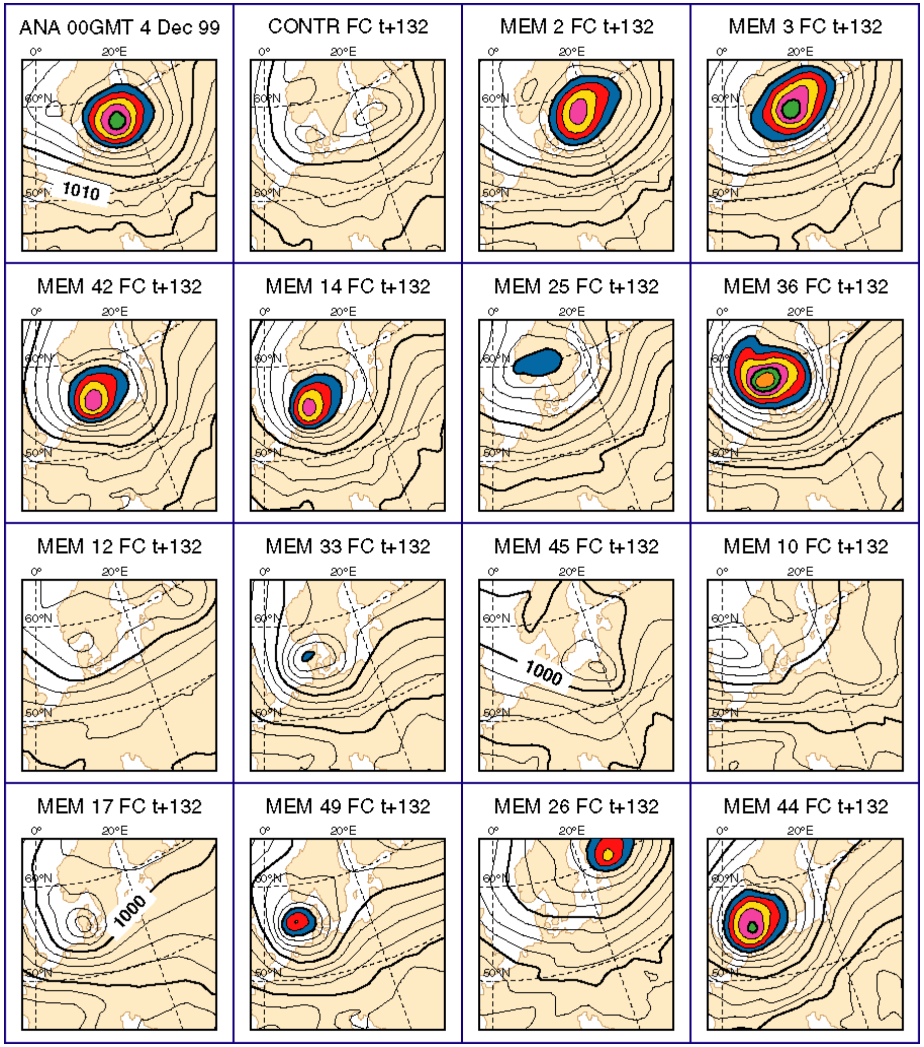
Weather forecasts lose skill because of the growth of errors in the initial conditions (initial uncertainties) and because numerical models describe the laws of physics only approximately (model uncertainties). As a further complication, predictability (i.e. error growth) is flow dependent. **The Lorenz 3D chaos model illustrates this.**



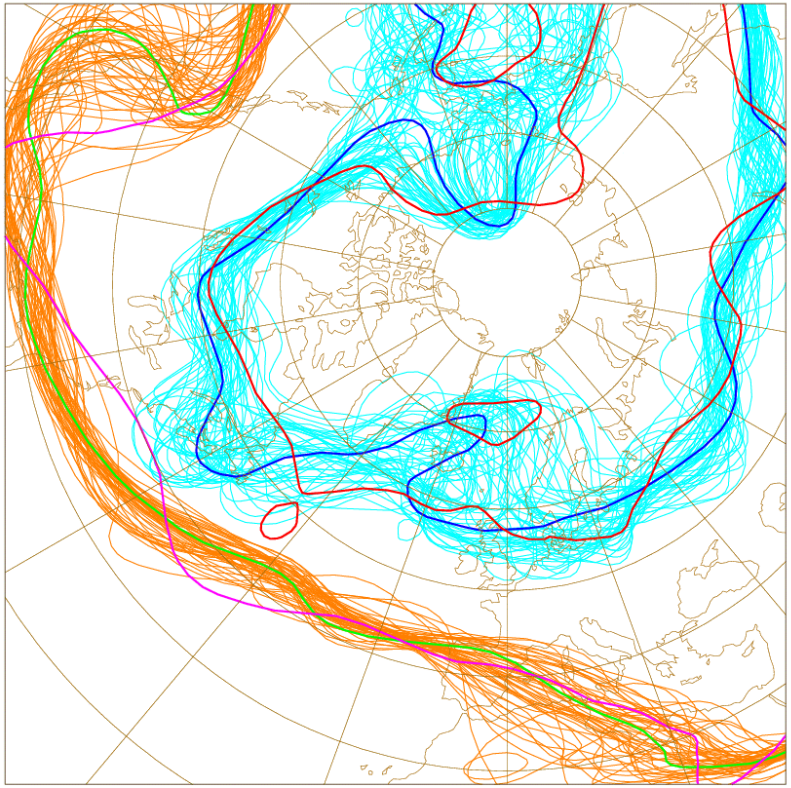
The atmosphere exhibits a chaotic behavior: an example

A dynamical system shows a chaotic behavior if most orbits exhibit sensitivity depends, i.e. if most orbits that pass close to each other at some point do not remain close to it as time progresses.

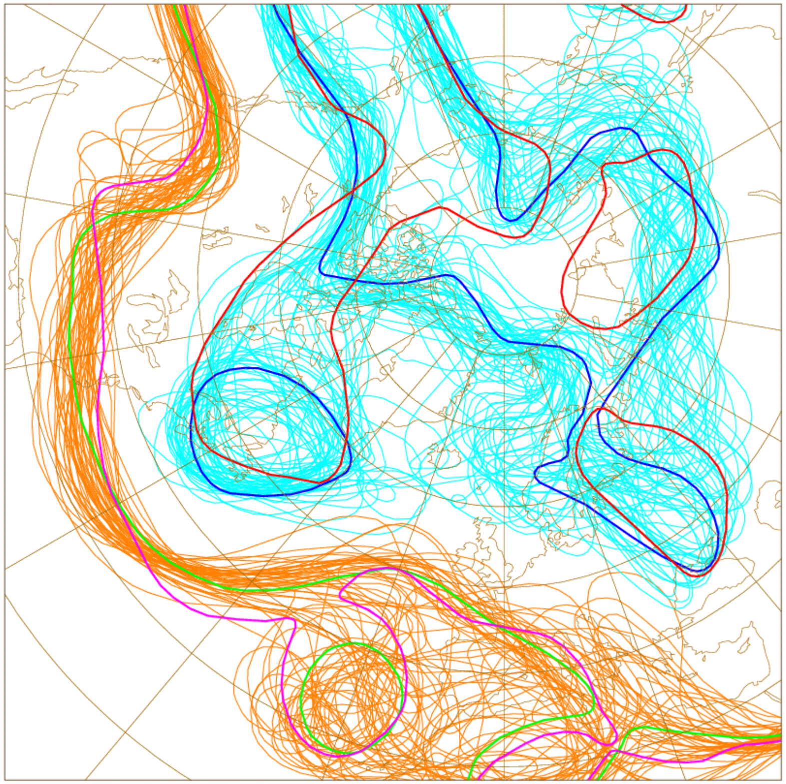
This figure shows the verifying analysis (top-left) and 15 132-hour forecasts of mean-sea-level pressure started from slightly different initial conditions (i.e. from initially very close points).



Predictability is flow dependent: spaghetti plots



500z d:1997-02-09 12:00:00 fc+120h cl:od exp:1
AN red/purple - CON blue/green - iso=5200-5700



500z d:1997-03-13 12:00:00 fc+120h cl:od exp:1
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The degree of mixing of Z500 isolines is an index of low/high perturbation growth.

The probabilistic approach to NWP: ensemble prediction

A complete description of the weather prediction problem can be stated in terms of the time evolution of an appropriate probability density function (PDF). Ensemble prediction based on a finite number of deterministic integration appears to be the only feasible method to predict the PDF beyond the range of linear growth.

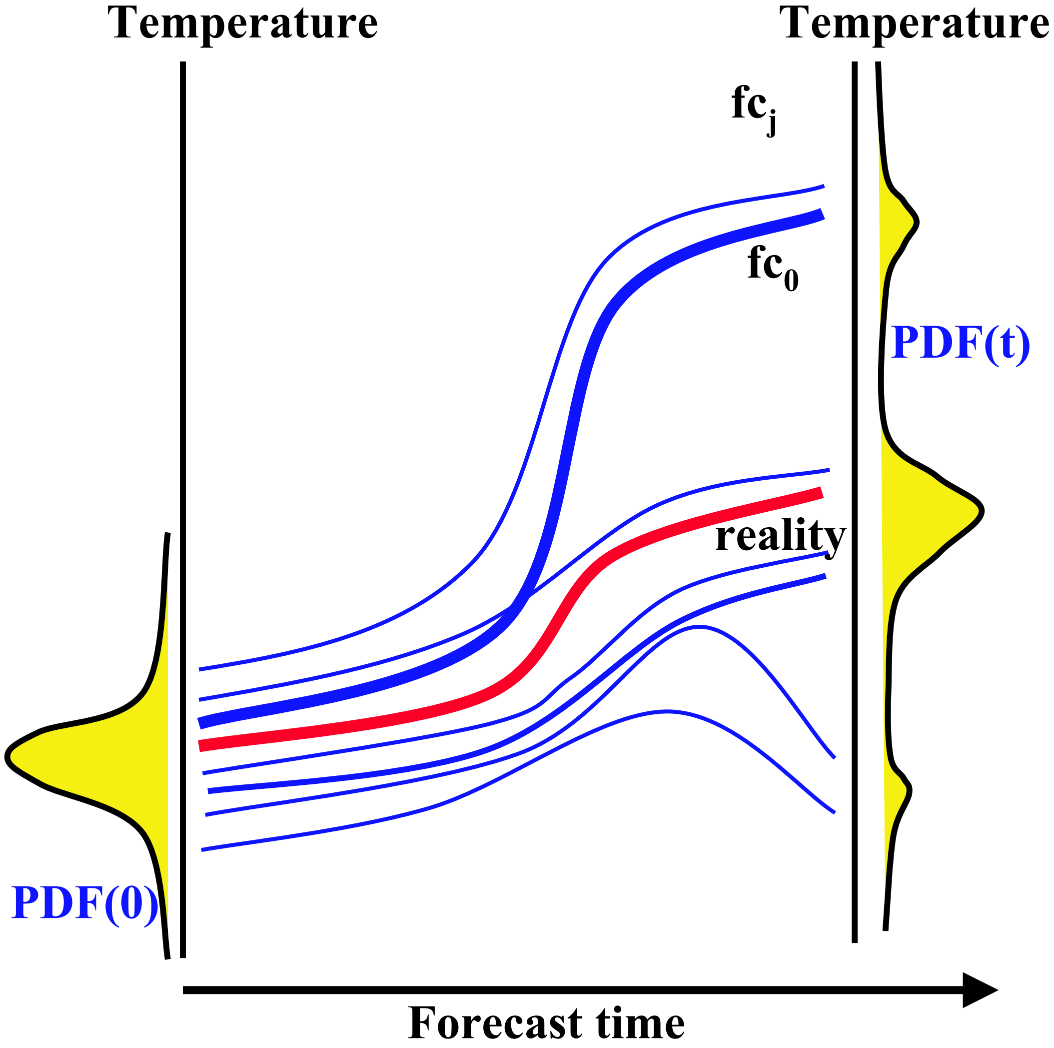
Currently, the ECMWF operational suite includes every day:

- a single deterministic 10-day forecast run at high resolution (TL511L60, 40km, 60 levels);
- 51 10-day forecasts run at lower resolution (TL255L40, 80km, 40 levels).

The 51 forecasts constitute the ECMWF Ensemble Prediction System. The first version of the EPS was implemented operationally in December 1992. The current version of the EPS simulates both initial and model uncertainties.

Schematic of ensemble prediction

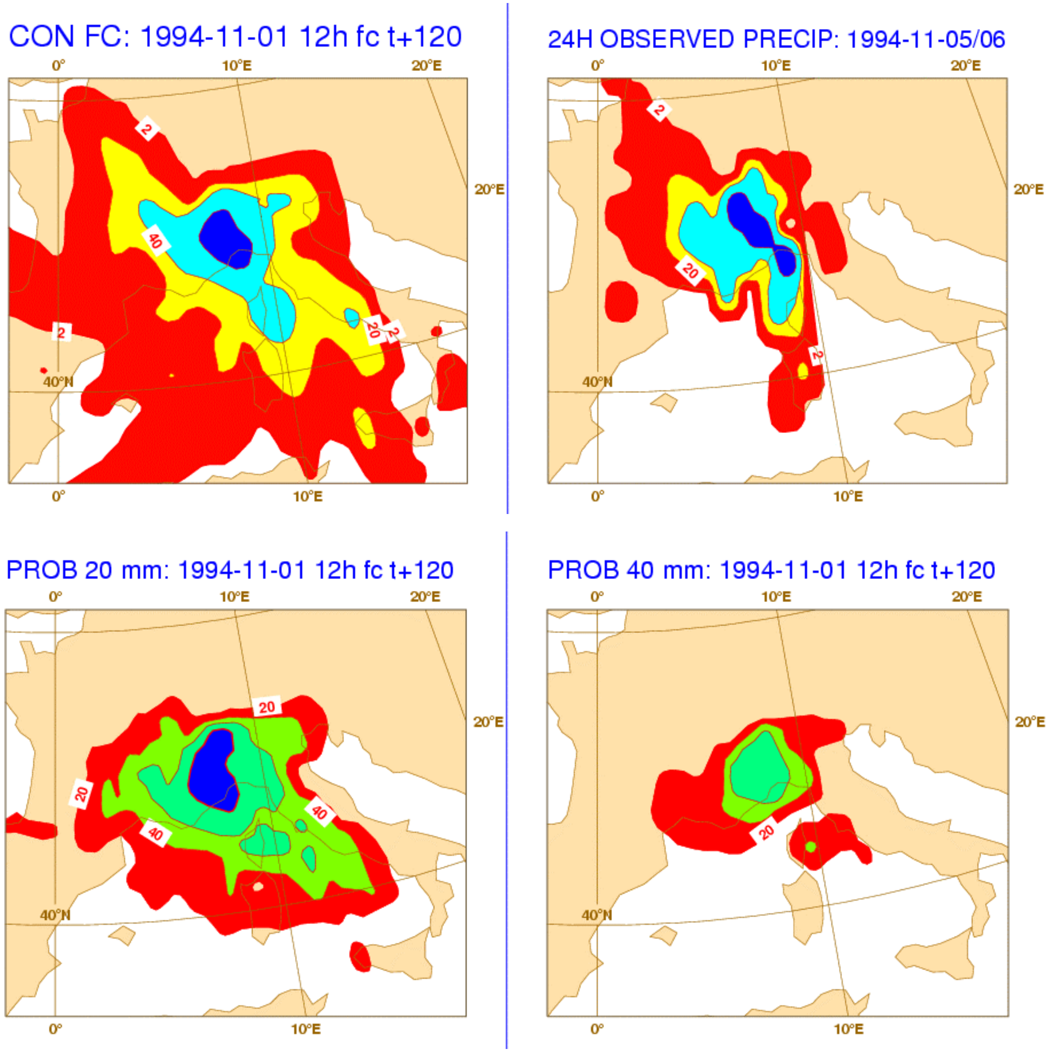
- Two are the main sources of error growth: initial and model uncertainties.
- Predictability is flow dependent.
- A complete description of weather prediction can be stated in terms of an appropriate probability density function (PDF). Ensemble prediction based on a finite number of deterministic integration appears to be the only feasible method to predict the PDF beyond the range of linear growth.



What does it mean to ‘predict the PDF time evolution’?

The EPS can be used to estimate the probability of occurrence of any weather event.

Floods over Piemonte (Italy), 6 Nov 94 (top right panel). The forecast skill of the single deterministic forecast given by the EPS control (top left) can be assessed by EPS probability forecasts (bottom panels).


CON FC: 1994-11-01 12h fc t+120

24H OBSERVED PRECIP: 1994-11-05/06

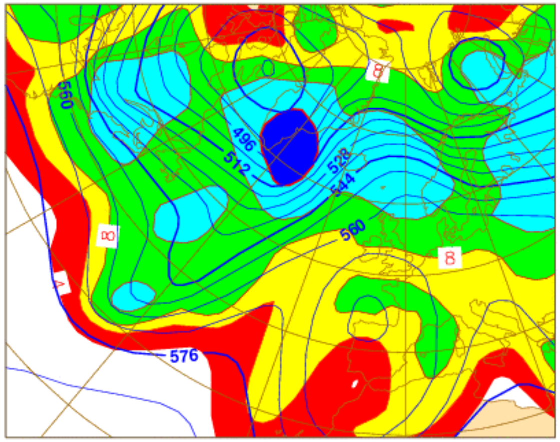
PROB 20 mm: 1994-11-01 12h fc t+120

PROB 40 mm: 1994-11-01 12h fc t+120

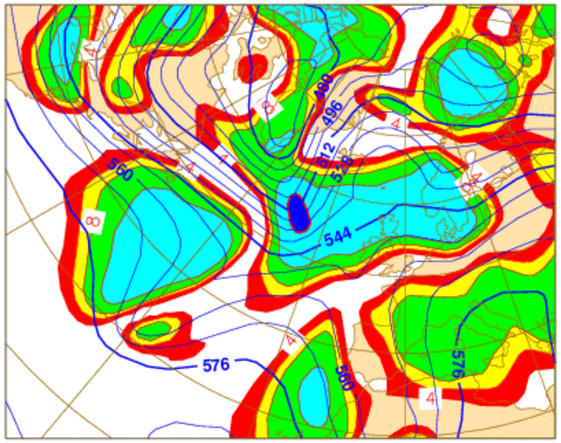
What does it mean to 'predict the PDF time evolution'?

The ensemble spread around the control forecast can be used to identify areas of potential large control-forecast error. These figures show the 5-day control forecast and ensemble spread (left) and the verifying analysis and the control error (right) for forecasts started 18 January 1997 (top) and 1998 (bottom).

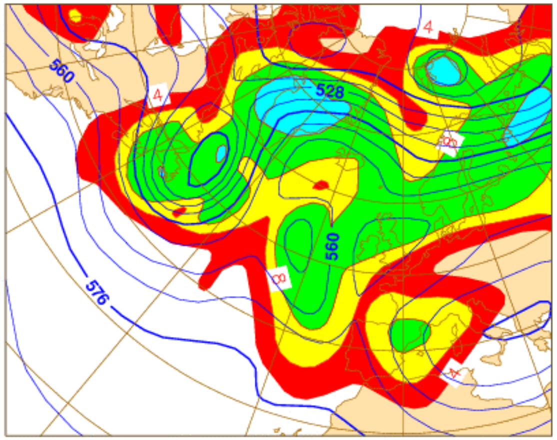
CON+SP - Z500 1997-01-18 12h fc t+120



ANA+ERRCON - Z500 1997-01-23 12h fc



CON+SP - Z500 1998-01-18 12h fc t+120



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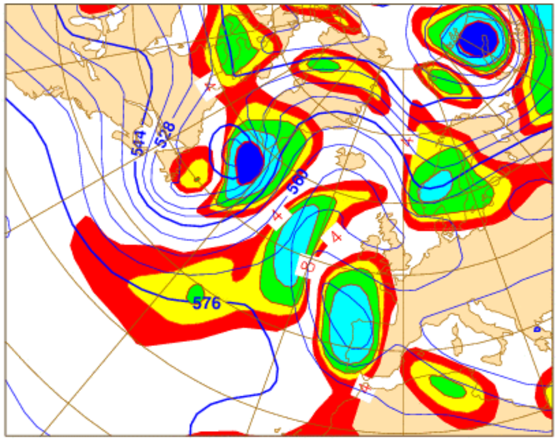
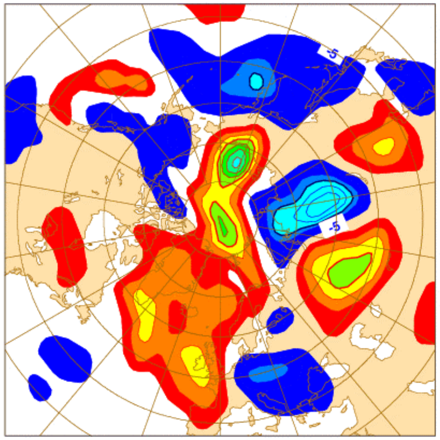


What should an ensemble prediction system simulate?

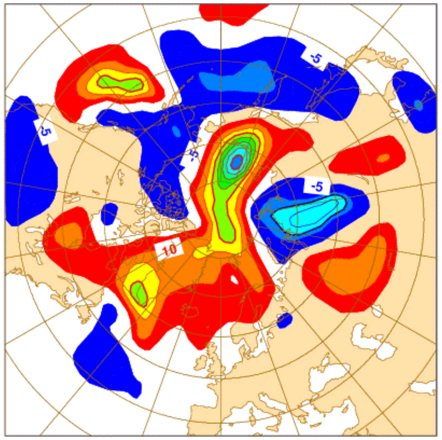
What is the relative contribution of initial and model uncertainties to forecast error?

Richardson **(1998, QJRMS) have compared forecasts run with two models (UKMO and ECMWF) starting from either the UKMO or the ECMWF ICs. Results have indicated that initial differences explains most of the differences between ECMWF-from-ECMWF-ICs and UKMO-from-UKMO-ICs forecasts.**

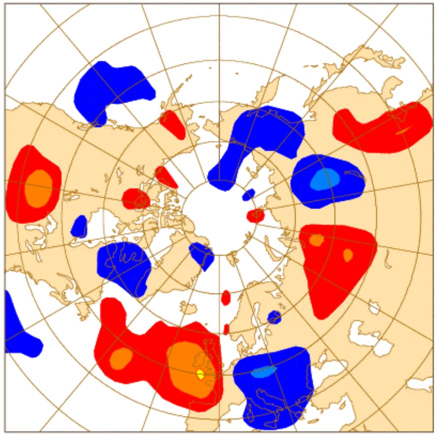
UK(UK)-EC(EC) Z500 1996-12-17 12h t+120



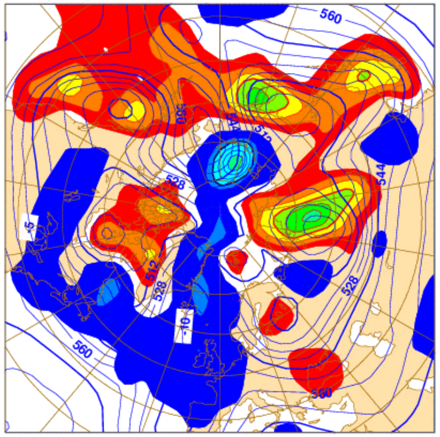
EC(UK)-EC(EC) Z500 1996-12-17 12h t+120



UK(UK)-EC(UK) Z500 1996-12-17 12h t+120



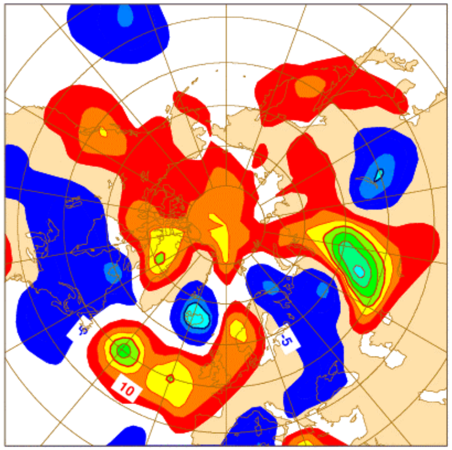
EC(EC)-ANA Z500 1996-12-17 12h t+120



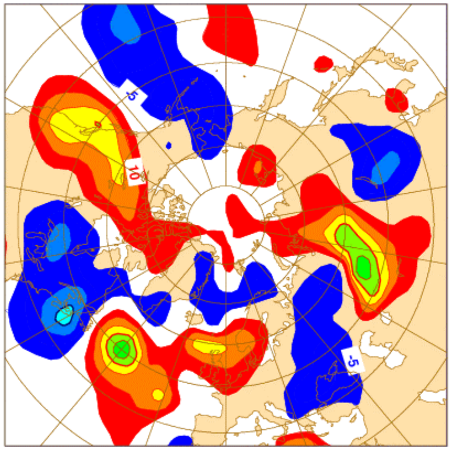
Initial uncertainties have a dominant effect

This figure shows the difference between 3 120-hour forecasts: UK(UK) (i.e. UK-from-UK-ICs) and EC(EC) (top left), EC(UK) and EC(EC) (top right), UK(UK) and EC(UK) (bottom left). The error of the EC(EC) forecast is also shown (bottom left). Initial differences contributes more than model differences to forecast divergence. This suggests that initial uncertainties contributes more than model approximations to error growth during the first 3-5 forecast days. **How should an ensemble prediction system simulate initial uncertainties?**

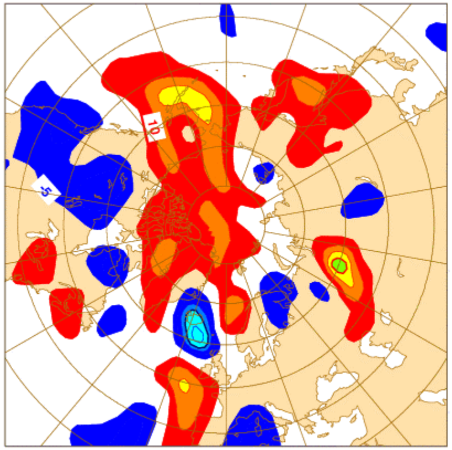
UK(UK)-EC(EC) - Z500 1996-12-22 12h t+120



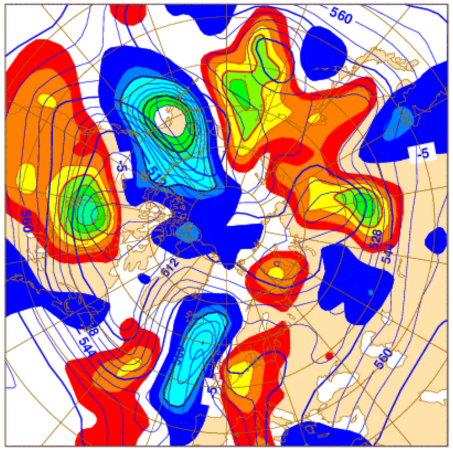
EC(UK)-EC(EC) - Z500 1996-12-22 12h t+120



UK(UK)-EC(UK) - Z500 1996-12-22 12h t+120



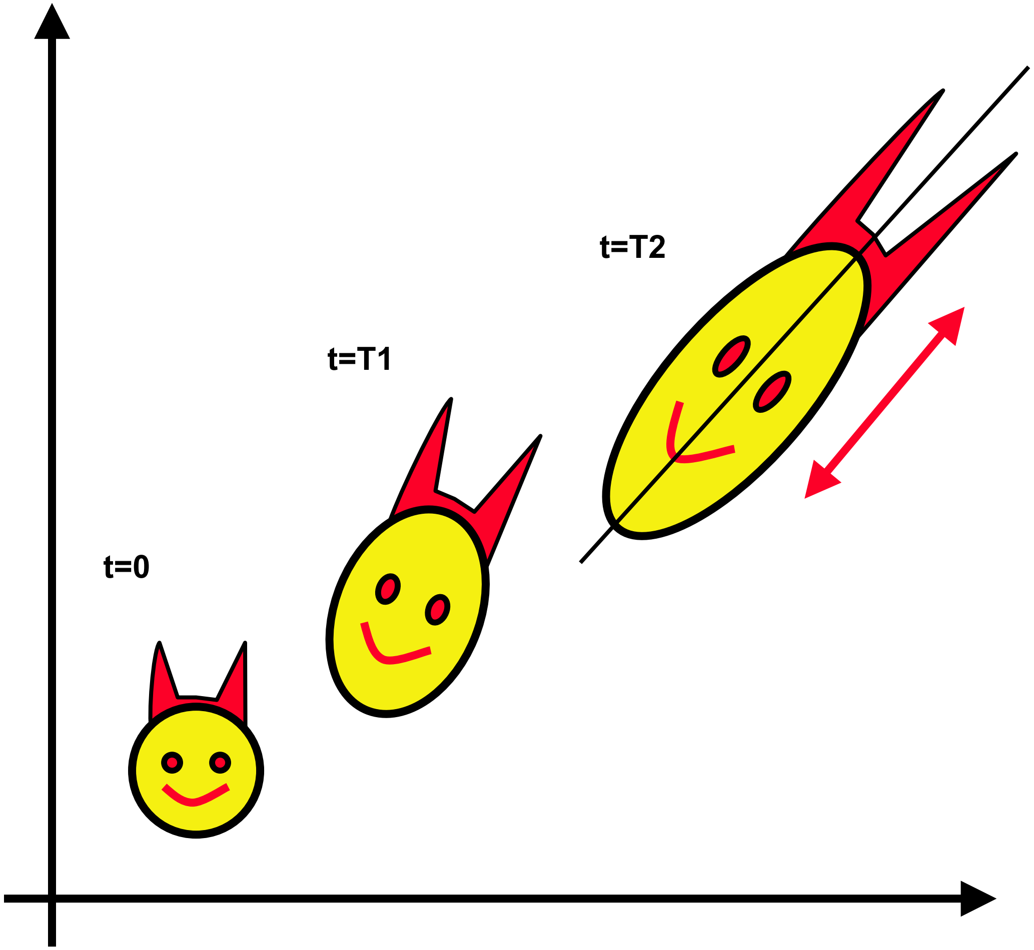
EC(EC)-ANA - Z500 1996-12-22 12h t+120



How should initial uncertainties be defined?

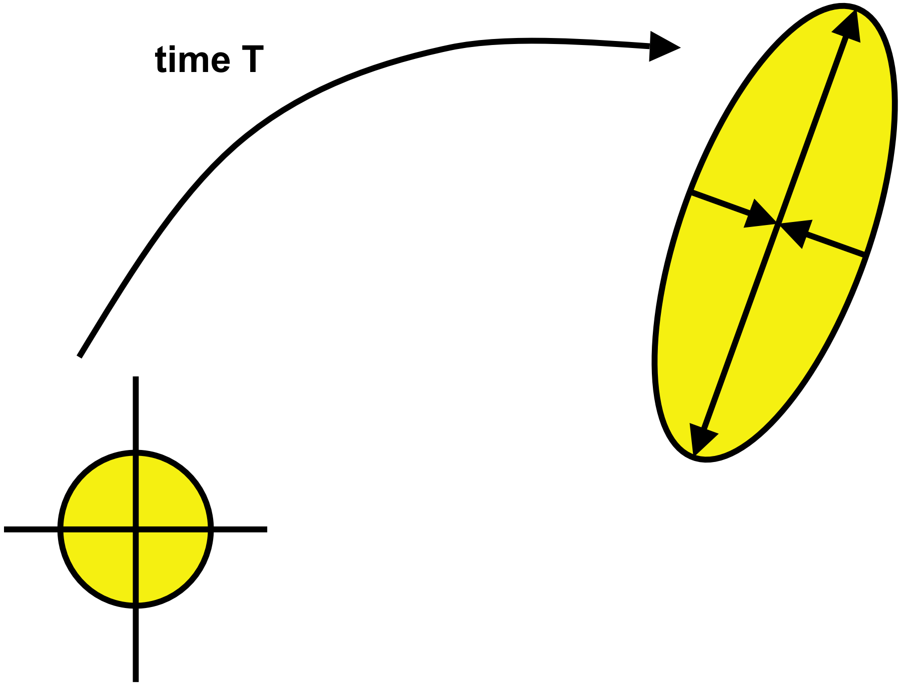
Perturbations pointing along different axes in the phase-space of the system are characterized by different amplification rates. As a consequence, **the initial PDF is stretched principally along directions of maximum growth**.

The component of an initial perturbation pointing along a direction of maximum growth amplifies more than a component along another direction (*Buizza et al* 1997).



Definition of the initial perturbations

To formalize the problem of the computation of the directions of maximum growth an **inner product (metric)** should be defined.



Inner product and norm definition

Given two state-vectors ***x*** and ***y*** expressed in terms of vorticity ζ, divergence ***D***, temperature ***T***, specific humidity ***q*** and surface pressure π, the following inner products (and the associated norms) can be defined (<..,..> is the Euclidean inner product)::

- total energy inner product (no humidity term):

$$< x; E_{TE} y >= \frac{1}{2} \iint (\nabla\Delta^{-1}\zeta_x \cdot \nabla\Delta^{-1}\zeta_y + \nabla\Delta^{-1}D_x \cdot \nabla\Delta^{-1}D_y + \frac{C_p}{T_r} T_x T_y) d\Sigma \frac{\partial p}{\partial \eta} d\eta$$

$$+ \int (R_d \frac{T_r}{p_r} \ln \pi_x \ln \pi_y) d\Sigma$$

- enstrophy inner product:

$$< x; E_{Ens} y >= \frac{1}{2} \iint (\nabla\Delta^{-1}\zeta_x \cdot \nabla\Delta^{-1}\zeta_y) d\Sigma \frac{\partial p}{\partial \eta} d\eta$$

- ψ-square inner product:

$$< x; E_{\psi^2} y >= \frac{1}{2} \iint (\psi_x \psi_y) d\Sigma \frac{\partial p}{\partial \eta} d\eta$$

Inner product and norm definition

Denote by $\zeta^{n,l}$ the level-l vorticity component with total wave number n, by $D^{n,l}$ …. of a state vector x.

The norm of ***x*** can be written in matrix form as:

$$\|x\|_{TE}^2 = \frac{1}{2}\sum_l \sum_n \begin{pmatrix} \zeta_x^{n,l} & D_x^{n,l} & T_x^{n,l} \end{pmatrix} \begin{pmatrix} -[R_a^2/n(n+1)]\delta p_l & 0 & 0 \\ 0 & -[R_a^2/n(n+1)]\delta p_l & 0 \\ 0 & 0 & [C_p/T_r]\delta p_l \end{pmatrix} \begin{pmatrix} \zeta_x^{n,l} \\ D_x^{n,l} \\ T_x^{n,l} \end{pmatrix}$$

$$+\sum_n R_d\, T_r/p_r \ln \pi_x^n \ln \pi_x^n$$

where n is the total wave number, δp is the pressure difference between two half-levels; T_r=350deg and p_r=100kPa are reference values; R_a=6371km, R_d=287JK^{-}kg^{-1}, C_p=1004JK^{-}kg^{-1}.

Definition of the system instabilities: normal modes

Consider an N-dimensional autonomous system:

$$\frac{\partial y}{\partial t} = A(y)$$

The method more commonly applied to study the stability of a solution **z** of the system equations is based on **normal modes**, whereby small disturbances are resolved into modes which may be treated separately because each of them satisfies the system equations. The system equations are linearized around the constant solution **z:**

$$\frac{\partial y}{\partial t} = A_l(z)y \qquad A_l(z) = \left.\frac{\partial A(z)}{\partial z}\right|_z$$

A normal mode is a solution of the linearized equations of the form:

$$y(x,t) = f(x)e^{\lambda t}$$

Definition of the system instabilities: normal modes

By substituting the normal mode definition into the linear equations an eigenvalue problem is defined:

$$A_l(z)f(x) = \lambda f(x)$$

The eigenvectors with real positive eigenvalues λ identify the **unstable normal modes** of the systems. A system is defined asymptotically stable if and only if every eigenvalue has negative real part.

Charney (1947) and *Eady* (1949) considered idealized atmospheric flows and by applying a normal-mode stability analysis they studied the **baroclinic instability** mechanism and showed that the zonal mean component of realistic mid-latitude flows is unstable. The resulting exponentially growing structure proved to have length and time scales similar to observed atmospheric cyclogenesis.

Asymptotic and finite-time instabilities

Farrell (1982) studying perturbations’ growth in baroclinic flows notices that the long-time asymptotic behavior is dominated by normal modes, but that **there are other perturbations that amplify more than the most unstable normal mode over a finite time interval**.

Farrell (1989) showed that perturbations with the fastest growth over a finite time interval could be identified solving an eigenvalue problem of the product of the tangent forward and adjoint model propagators. This result supported earlier conclusions by *Lorenz* (1965).

Calculations of perturbations growing over finite-time interval intervals have been performed, for example, by *Borges & Hartmann* (1992) using a barotropic model, *Molteni & Palmer* (1993) with a quasi-geostrophic 3-level model, and by *Buizza et al* (1993) with a primitive equation model.

Singular vector definition: the linear equations

Consider an N-dimensional autonomous system:

$$\frac{\partial y}{\partial t} = A(y)$$

Denote by ***z'*** a small perturbation around a time-evolving trajectory ***z***:

$$\frac{\partial z'}{\partial t} = A_l(z)z' \qquad\qquad A_l(z) = \left.\frac{\partial A(z)}{\partial z}\right|_z$$

$$\frac{\partial z}{\partial t} = A(z)$$

The time evolution of the small perturbation ***z'*** is described to a good degree of approximation by the linearized system ***$A_l(z)$*** defined by the trajectory. Note that the trajectory is not constant in time.

Singular vector definition: the linear propagator

The perturbation ***z'*** at time ***t*** is given by the time integration from the initial state ***z'(t=0)*** of the linear system:

$$z'(t) = z'_0 + \int_0^t A_l(z)\,d\tau$$

The solution can be written in terms of the **linear propagator *L(t,0)***:

$$z'(t) = L(t,0)z'_0$$

The linear propagator is defined by the system equations and depends on the trajectory characteristics. The E-norm of the perturbation at time ***t*** is given by:

$$\|z'(t)\|^2 = < z'(t); Ez'(t) > = < L(t,0)z'_0; EL(t,0)z'_0 >$$

Singular vector definition: the adjoint operator

Given any two vectors ***x*** and ***y***, the adjoint operator $\boldsymbol{L}^*$ of the linear operator ***L*** with respect to the Euclidean norm <..,..> is the operator that satisfies the following property:

$$< L^* x; y >=< x; Ly >$$

Using the adjojnt operator $\boldsymbol{L}^*$ the time-t E-norm of z’ can be written as:

$$\left\| z'(t) \right\|^2 =< Lz'_0; ELz'_0 >=< z'_0; L^* ELz'_0 >$$

Singular vector definition: the problem

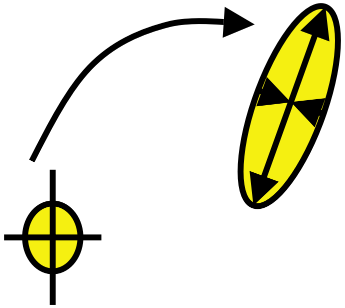
The problem of the computation of the directions of maximum growth can be stated as ‘finding the directions in the phase-space of the system characterized by the maximum ratio between the time-t and the initial norms’. Formally, this problem reduces to an eigenvector problem:

$$\max_{x_0 \in \Sigma} \frac{\|x(t)\|_E^2}{\|x_0\|_E^2} = \max_{x_0 \in \Sigma} \frac{< x_0 ; L^* E L x_0 >}{< x_0 ; E x_0 >}$$

The problem can be generalized by using two different norms at initial and final time:

$$\max_{x_0 \in \Sigma} \frac{\|x(t)\|_E^2}{\|x_0\|_{E_0}^2} = \max_{x_0 \in \Sigma} \frac{< x_0 ; L^* E L x_0 >}{< x_0 ; E_0 x_0 >}$$

Singular vector definition: the eigenvalue problem



Apply the following coordinate transformation:

$$y = E_0^{1/2} x$$

Then the generalized problem reduces to:

$$\max_{x_0} \frac{\left\| x(t) \right\|_E^2}{\left\| x_0 \right\|_{E_0}^2} = \max_{y_0} \frac{< y_0 ; E_0^{-1/2} L^* E L E_0^{-1/2} y_0 >}{< y_0 ; y_0 >}$$

The directions of maximum growth are defined by the following eigenvalue problem:

$$E_0^{-1/2} L^* E L E_0^{-1/2} \nu = \sigma^2 \upsilon$$

Singular vector definition: the eigenvalue problem

Define:

$$K = E^{1/2} L E_0^{-1/2}$$

$$K^* = (E^{1/2} L E_0^{-1/2})^* = E_0^{-1/2} L^* E^{1/2}$$

Then the eigenvalue problem can be written as:

$$(K^* K)\nu = \sigma^2 \upsilon$$

The eigenvectors of ***K*K*** and the corresponding eigenvalues are called the **singular vectors** and **singular values of K**. Note that the problem is defined by the metrics ***E*** and $\boldsymbol{E_0}$ and by the tangent forward propagator ***L*** (the adjoint operator $\boldsymbol{L^*}$ is defined by the linear).

Singular vector definition: the linear propagator

The **linear propagator** is defined by the model equations. Currently, the propagator can be schematically written as the product of the following operators:

$$L(t,0) = [\prod_{j=1}^{N_t} L_{phys}(t_j, t_{j-1}) L_{dyn}(t_j t_{j-1})] L_{NMI}$$

where the time integration between the initial time t=0 and the final time ***t*** has been discretized in $\boldsymbol{N_t}$ time steps, and where:

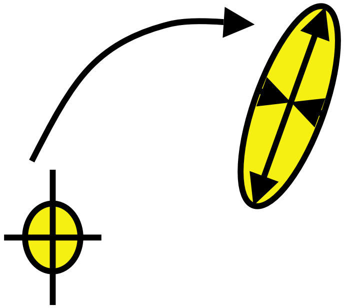
- $\boldsymbol{L_{phys}}$ represents the parameterized physical processes;
- $\boldsymbol{L_{dyns}}$ represents the adiabatic processes and horizontal diffusion;
- $\boldsymbol{L_{NMI}}$ represents the normal-mode initialization procedure.

The time interval t is called the **optimisation time interval**.

Singular vector definition: summary

In summary, the problem of the computation of the directions of maximum growth of a time evolving trajectory reduces to the computation of the **singular vectors** of **$K=E^{1/2}LE_0^{-1/2}$**, i.e. to solving the following eigenvalue problem:

$$E_0^{-1/2}L^{*}ELE_0^{-1/2}\nu=\sigma^{2}\upsilon$$



By definition, the singular vectors depend:

- on the **initial and final time metrics *E_0*** and ***E*;**
- on the **linear propagator *L(t,0)***;
- on the **time-evolving trajectory** along which they are computed**;**
- on the **optimization time interval.**

Conclusion

- Initial and model uncertainties are the main sources of error growth. Initial uncertainties dominates during the first 3-5 forecast days. Predictability is flow dependent.
- A complete description of weather prediction can be stated in terms of an appropriate probability density function (PDF). Ensemble prediction based on a finite number of deterministic integration appears to be the only feasible method to predict the PDF beyond the range of linear growth.
- The initial error components along the directions of maximum growth contribute most to forecast error growth.
- A metric has been defined to measure ‘forecast error growth’. The time evolution of small perturbations can be described in a linear approximation. The adjoint operator has been defined.
- The directions of maximum growth (singular vectors) are defined by an eigenvalue problem.

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