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# A data based mechanistic approach to nonlinear flood routing and adaptive flood level forecasting

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ABSTRACT

Operational flood forecasting requires accurate forecasts with a suitable lead time, in order to be able to issue appropriate warnings and take appropriate emergency actions. Recent improvements in both flood plain characterization and computational capabilities have made the use of distributed flood inundation models more common. However, problems remain with the application of such models. There are still uncertainties associated with the identifiability of parameters; with the computational burden of calculating distributed estimates of predictive uncertainty; and with the adaptive use of such models for operational, real-time flood inundation forecasting. Moreover, the application of distributed models is complex, costly and requires high degrees of skill. This paper presents an alternative to distributed inundation models for real-time flood forecasting that provides fast and accurate, medium to short-term forecasts. The Data Based Mechanistic (DBM) methodology exploits a State Dependent Parameter (SDP) modelling approach to derive a nonlinear dependence between the water levels measured at gauging stations along the river. The transformation of water levels depends on the relative geometry of the channel cross-sections, without the need to apply rating curve transformations to the discharge. The relationship obtained is used to transform water levels as an input to a linear, on-line, real-time and adaptive stochastic DBM model. The approach provides an estimate of the prediction uncertainties, including allowing for heterescadasticity of the multi-step-ahead forecasting errors. The approach is illustrated using an 80 km reach of the River Severn, in the UK.



## 1. Introduction

The main goal of a flood forecasting system, such as the National Flood Forecasting System of the UK Environment Agency (see e.g. [1,2]), is to provide reliable guidance for decisions on the initiation of flood warning and emergency procedures, with as long a lead-time as possible before the flood event. Long lead-time forecasts are required in early flood alert systems but, as an event progresses, more accurate forecasts with shorter lead times are required for warning purposes. As forecast reliability necessarily decreases with increasing forecast lead times, some compromise must be reached in order to minimise losses due to flooding, while avoiding the loss of trust and the material costs resulting from emergency procedures implemented unnecessarily. An operational flood forecasting system should, therefore, provide forecasts ranging from long, through medium, to short lead times. The length of the forecast depends on the available information upstream, which may include the observations of water levels (forming a basis for a short-term forecast) and/or rainfall measurements (for medium term forecasts). When meteorological ensemble forecasts are available they may be used to extend further the forecast lead, thus giving a long-term forecast [3,4]. Another important requirement of forecasting system is providing the forecast uncertainty (see e.g. [5]).

Operational flood forecasting systems, such as those reviewed, for example, in [6], include three major elements: a real time data acquisition system; hydrological and/or hydraulic models for simulation; and a system for updating. This paper is a sequel to an earlier paper [7] that considered a catchment model and associated forecasting system, including nonlinear rainfall-level and *linear* level (stage) routing models. The present paper will focus mainly on *nonlinear* level routing tools and their use within the forecasting

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system. We believe that models should be fit for purpose and we shall demonstrate that the proposed approach fulfils the requirements of an operational flood forecasting system.

Channel flow routing models used in on-line flood forecasting span a range from distributed physically-based models [8,9] to models based on Artificial Neural Networks [10–12] and nonlinear time series analysis [13,14]. Among the distributed models for flow routing the most popular are ISIS, Mike11, Lisflood, HecRas and Sobek (see e.g. [15–18]). Computer intensive ensemble methods, including adaption, are also being developed (e.g. [19–21]), although these methods are not included in popular hydrological software. However, all these models are deterministic and even though it is possible to apply them in a stochastic framework (e.g. [22]), such a framework is challenging for operational flood forecasting [6]. The so-called probabilistic forecasts produced when ensemble meteorological forecasts are used [15,16] do not incorporate either the parametric or the observational uncertainty associated with the flow routing models.

On-line updating and data assimilation procedures are another important part of operational flood forecasting. The methods include input updating, parameter/state updating, output/error updating [23,6]. Such updating procedures use real-time observations of hydrological and/or meteorological variables to improve model performance. The most common of these are state updating using Kalman filter [24,19,20] and error updating, where the error series is modelled in stochastic terms and this is used to improve the forecasts (e.g. [25] and references therein).

It should be noted that the use of efficient updating procedures, such as Kalman filtering, is not easily accommodated in complex, distributed flow routing models [25,26]. Moreover, the use of such increasingly complex methods can be disadvantageous in operational use because a flood forecasting system may have to be applied by people who possess a great deal of empirical experience but do not necessarily possess the knowledge and experience required to handle models of such complexity. For this reason, we believe that there is still a role for simple but effective methods that are more transparent to the user.

The approach to on-line data assimilation and forecasting utilising linear Data-Based Mechanistic (DBM) modelling followed in the present paper combines state updating using Kalman filter with output updating via on-line recursive estimation of the nonlinear gain. The methodology was introduced by [27] and applied in the context of operational system forecasting in [7]. The methodology applies stochastic models and, therefore, provides forecasts together with uncertainty estimates. This modelling tool can be incorporated into DELFT-FEWS flood forecasting system [2] which has a modular structure, and forms a kind of interface that controls the flow of information between three elements of the forecasting system and the user (flood warning centres). Apart from the ability to include a very wide range of modelling tools provided by the user, it also uses a standard error correction module. However, it cannot provide a universal framework for uncertainty analysis for all types of models, which has to be produced by the inserted flow routing model.

The general approach used in the present paper has been applied to the 157 km length the River Severn in the UK, between Abermule and Bewdley, as shown in Fig. 1 (see [7,28]). Within this reach there is one major tributary input from the Vrnwy catchment, and a number of minor ungauged tributaries. The flood forecasting system consists of a number of reaches, with the forecast lead-time at each of the gauging stations extended by the forecasts produced for the more upstream reaches. The system is aimed at providing long-term forecasts (up to 35 h for the case study presented) using a simplified data assimilation and uncertainty propagation system. Forecast accuracy in this study was limited by presenting the results only for the longest forecast lead time. However, as would be expected from hydraulic principles, the results indicate that the celerity of the flood wave depends on the channel and floodplain geometry, such that a routing model based on linear dynamics cannot give equally good predictions for both high and low flows.

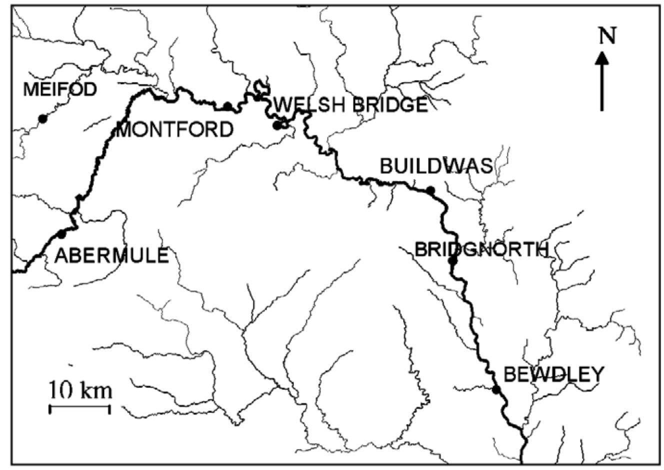


**Fig. 1.** Gauging stations in the middle reach of the River Severn, UK.

The present paper presents a novel solution to medium-term flood wave routing aimed at flood forecasting with lead times equal to, or shorter than, the natural delays in the system (up to 25 h in the case study presented later in Section 3 of the paper). The novelty of the approach is its use of a nonlinear transformation of the water levels at the input to a river reach, in order to represent this nonlinear routing process between the gauging stations, and the serial connection of a Stochastic Transfer Function (STF) to describe the linear dynamic relationship between the transformed input and output (so that the complete reach model is of a similar generic form to the nonlinear rainfall-flow models identified in our earlier papers cited above: namely the 'Hammerstein' type model: see also [29]).

The idea of nonlinearity in rainfall-flow and flow routing relationships is not new and dates back to the laboratory experiment described by [30], who demonstrated that rainfall–runoff process is nonlinear. Among others, further work on nonlinear conceptual models was presented by [31] (see also [32,33]) who introduced a cascade of nonlinear conceptual reservoirs. The approaches presented in these papers and other works of a similar kind follow the reductionist approach – i.e. the scientist starts from the full process description and reduces it down to the dominant features that are most important for a given application. However, the DBM approach advocated here takes an opposite route; namely, the model structure is first identified from the data prior to the estimation of the model parameters and their subsequent interpretation in physical terms [27]. Therefore, the choice of model parameters (the statistical model identification step) is based on a data-based, inductive process that is conditioned on the physically interpretation. This data-based model identification step means that, normally, the model will have the simplest identifiable model structure within the generic class of models being considered (here the discrete-time equivalent of ordinary differential equation models).

One of the main differences between a model obtained by DBM modelling and the reductionist approach lies in the way the nonlinearity of the process is specified. In DBM modelling, it is identified directly from the available observations, which normally provides improved forecast accuracy over the range of water levels considered in the modelling analysis. Nonlinear transformation of upstream water levels allows for the separation of a static nonlinearity from the linear process dynamics. In other words, we look for the static nonlinear transformation of input (water levels at a cross section upstream) which would lead to the linear form of the main process dynamics. Also, we restrict the search for the description of process dynamics to a class of linear models, which are physically realisable (i.e. have real roots). The decomposition of the process into nonlinear static and linear dynamic parts is identified in a reasonably objective manner using State Dependent Parameter (SDP) estimation (see next Section 2). As a result, the type of models we obtain is different from the cascade of nonlinear conceptual reservoirs presented by [31], which do not allow for the separation of nonlinearity from the linear dynamics.

The adaptive flood routing methodology presented in this paper is demonstrated on the 80 km Welsh Bridge to Bewdley section of the River Severn. The transformed input water levels are used as an input to an on-line data assimilation system, similar to that introduced in [7]. The method gives very good results for forecast lead times within the natural delays in the system. It relies on data acquisition along the river at as many points as is economically justified, in order to obtain distributed forecasts along the river.

## 2. Methodology

The proposed separation of nonlinear transformation and the linear dynamic routing for a single river reach is shown schematically in Fig. 2. This model form is not assumed *a priori* but is identified from the measured data by an inductive process of Data-Based Mechanistic (DBM) modelling (see e.g. [34] and references therein). More specifically in the present context, it is identified as a nonlinear State Dependent Parameter (SDP) transformation of water levels from the cross-section upstream to that downstream (where the SDP nonlinearity is thought primarily to be a function of the geometry of the reach), in series with a model for linear dynamic dispersion along the river reach in the form of a Stochastic Transfer Function (STF) model, with a pure time delay $\delta$ introduced to account for the advective delay related to the celerity of the flood wave. It is possible that the nonlinearity is also accounting for small changes in this time delay. Using the normal DBM approach, which requires a physical interpretation of the estimated model, this linear STF can be considered as representing one or more (depending on the identified order of the STF) linear, dynamic storage elements. The interpretation of such STF models in these terms is discussed in some detail by [35].

### 2.1. The DBM model and forecasting system

In the case of discrete-time, sampled data at a uniform sampling interval of $\Delta t$ time units (here, 1 h intervals), the SDP transformation of water levels has the following form:

$$\tilde{u}_k = g(u_k) \cdot u_k, \quad (1)$$

where $\tilde{u}_k$ is the transformed upstream water level at the $k$th sampling instant and $g(\cdot)$ denotes the associated nonlinear function.

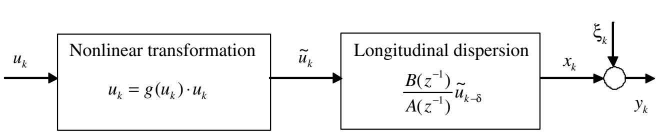


**Fig. 2.** Schematic diagram of the nonlinear, discrete-time, DBM level routing model: the first block denotes the SDP based nonlinear transformation of water levels, while the second block describes the STF-based linear model dynamics; the polynomials $A(z^{-1})$ and $B(z^{-1})$ in the backward shift operator are defined in the explanation of Eq. (2), with the denominator polynomial $A(z^{-1})$ having real roots (see text).

The SDP method of analysing relationships such as this exploits an approach to non-stationary and nonlinear signal processing based on the identification and estimation of stochastic models with state dependent parameters, as introduced in a number of previous references [36,37]. As we shall see later, the SDP relationship is first estimated in a non-parametric (graphical) manner, in order to identify the shape of the nonlinearity. Subsequently, the obtained nonlinear relationship in the form of a nonlinear SDP gain is parameterised with the help of a user-defined parametric form. This parameterised model can be defined in any suitable manner: e.g. a combination of polynomial, exponential and piecewise linear analytical relations; or the use of more general parameterisations, such as radial basis functions (e.g. [38]). In defining the parameterisation, however, care should be taken to ensure that it will perform satisfactorily in an extrapolative sense: i.e. if future water levels are encountered outside the range of levels represented in the estimation and validation data.

At the reach scale, the parameterised SDP nonlinearity provides the 'effective water level' input into a dynamic dispersion model identified as a linear, discrete-time STF, i.e.

$$x_k = \frac{B(z^{-1})}{A(z^{-1})}\tilde{u}_{k-\delta}, \quad y_k = x_k + \xi_k, \quad (2)$$

where $x_k$ is the underlying 'true' level, $y_k$ is the noisy observation of this level, $\delta$ is a pure, advective time delay of $\delta\Delta t$ time units, while $A(z^{-1})$ and $B(z^{-1})$ are polynomials of the following form:

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n};$$
$$B(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_m z^{-m}$$

in which $z^{-r}$ is the backward shift operator, i.e. $z^{-r}y_k = y_{k-r}$, and $A(z^{-1})$ is assumed to have real roots (eigenvalues) that lie within the unit circle of the complex $z$ plane. The additive noise term $\xi_k$ in (2) is usually both heteroscedastic (i.e. its variance changes over time) and autocorrelated in time. It is assumed to account for all the uncertainty *at the output* of the system that is associated with the inputs affecting the model, including measurement noise, unmeasured inputs, and uncertainties associated with the model structure. The orders of the polynomials $n$ and $m$ are identified from the data during the data-based identification process and are normally in the range 1–2 (in the later example, for instance, $n = m = 1$). In the following analysis, the triad $[n \quad m \quad \delta]$ is used to characterize this model structure.

A block diagram of the model (2) for a single river reach is shown in Fig. 2, where it should be noted that the linear transfer function in the second block is not a 'black-box' model. This is because its denominator $A(z^{-1})$ is constrained to have real roots, so that the transfer function can be decomposed into a serial or parallel connection of first order processes and, as pointed out previously, these can be interpreted in physically meaningful terms as linear, dynamic storage elements that explain the dispersive dynamics of the reach. The nonlinear transformation of the input variable (water level upstream) represents the total level-routing nonlinearity in this model. So far, we have not been able to fully explain this nonlinearity in physically meaningful terms since it could be due to a number of factors, including the cross-sectional geometry of the river and its (possibly complex) variation in form between the gauged sites. In any case, the physical interpretation would probably be better addressed in terms of the flow-routing equivalent of this model, where mass conservations considerations could be taken into account, and research is proceeding on this topic.

The parameters of the complete reach model are obtained by statistical identification and estimation analysis based on the observed water level data at locations along the river. Initially, the

Refined Instrumental Variable (RIV) algorithm [39] is used to identify and estimate the best *linear* STF model between two adjacent locations. The identified structure of this linear model (the triad $[n \quad m \quad \delta]$) then provides the basis for the preliminary, non-parametric identification of the nonlinear SDP transfer function relationship. In the present context, this normally identifies a first order $[1 \quad 1 \quad \delta]$ relationship, where only the single numerator parameter $b_0$ in $B(z^{-1})$ is state-dependent. The estimate $\hat{b}_0(u_k)$ of this SDP then defines directly the non-parametric estimate of the SDP input nonlinearity. The improved RIV estimation of the STF model parameters follows using the upstream level input transformed by this non-parametric nonlinearity (see previous references). This combination of the non-parametric SDP nonlinearity and the STF model constitutes the *a priori* identification of the complete nonlinear DBM reach model. It provides the basis for the final model estimation analysis, where the SDP nonlinearity is parameterized in an appropriate manner (see above and later) and all the parameters in this parameterized nonlinear function, as well as the STF model parameters, are finally optimized concurrently using a nonlinear least squares optimization algorithm.

It is important to note that the DBM model, identified and estimated in the above manner, efficiently reflects the information content of the data, so that the possibility of over-parameterization and associated poor identifiability is minimized. The additional parameters introduced by the parameterisation of the SDP nonlinearity should not lead to over-parameterisation of the model because they do not affect its dynamic order, which is the critical factor in terms of identifiability, and are normally well identified statistically from the data.

Having identified and estimated the model from the available calibration data, it needs to be converted to a form that is appropriate for forecasting applications. Here, the model (2), with its input $\tilde{u}_k$ defined by Eq. (1), is reformulated in state space terms and applied within an on-line Kalman filter data assimilation and forecasting engine, following the procedure described in [27,7]. For analytical purposes connected with the use of the Kalman filter, the additive noise $\xi_k$ in the TF model (2) is assumed to be zero mean, serially uncorrelated white noise with variance $\sigma_k^2$, where the subscript $k$ allows for the presence of the heteroscedasticity. The Kalman filter also requires the introduction of input 'system' noise $\eta_k$ which, like $\xi_k$, is considered as zero mean, serially uncorrelated white noise, this time with variance $\sigma_\eta^2$. In the present context, this input noise accounts for the uncertainty in the upstream input level and its effect on the transformed input $\tilde{u}_k$. With these stochastic inputs defined and quantified in the manner discussed later, the Kalman filter, generates the level forecasts with associated 95% confidence bounds.

In order to account for small scale modelling errors, the model also includes an on-line gain updating procedure that is built into the system using real-time recursive estimation [27,39]. This involves the adaptive update of a time-variable gain factor $p_k$ in the expression:

$$y_{k+f} = \hat{p}_k \cdot \hat{x}_{k+f|k} + \eta_{k+f}. \tag{3}$$

Here, $\hat{x}_{k+f|k}$ is the $f$-step-ahead water level forecast based on the data received up to the $k$th sampling instant and $\eta_k$is a noise term that represents the error in the forecast; $\hat{p}_k$ denotes an estimate of $p_k$, which is assumed to vary stochastically as a Random Walk (RW) process with a hyper-parameter $q_k$, that defines the variance of the stochastic white noise input to the RW model, optimized to yield good forecasting performance (see below and the cited references). This allows the parameter estimate $\hat{p}_k$ to change over time when it is estimated in real-time using a time variable parameter Recursive Least Squares (RLS) algorithm [39], conditioned on the observations up to time $k$.

The uncertainty bounds on the forecasts are derived assuming that forecast variance is a function of estimated water levels [28]. In particular, the observation noise variance is identified from the data to be heteroscedastic, with the temporal changes induced by a dependence on the underlying stochastic water level $x_k$. This is parameterised in terms of the estimate $\hat{x}_k$ of the level, with the form:

$$\hat{\sigma}_k^2 = \sigma_0^2 + \sigma_1^2 \hat{x}_k^2. \tag{4}$$

The $f$-step ahead forecast variance is then obtained by augmenting (4) by the variance related to the $f$-step ahead state error covariance, following [7]: i.e.,

$$\mathrm{var}(\hat{x}_{k+f|k}) = \hat{\sigma}_k^2 + \mathbf{h}^T \mathbf{P}_{k+f|k} \mathbf{h}, \tag{5}$$

where $\mathbf{P}_{k+f|k}$ is the error covariance matrix estimate associated with the $f$-step ahead prediction of the state estimates; and $\mathbf{h}$ denotes the observation vector (see previously cited references). The resulting $f$-step-ahead prediction variance is used to derive approximate 95% confidence bounds for the forecasts, under the assumption that the uncertainty will be dominated by the first two statistical moments associated with the probability distribution of $\hat{x}_{k+f|k}$.

Note that, for a first order STF model of the kind identified in the later example, the above formulation is particularly simple, since the state space representation of (2) has just one state variable, namely the underlying stochastic water level $x_k$, i.e.,

$$\begin{aligned} x_k &= a_1 x_{k-1} + b_0 \tilde{u}_k + \eta_k, \\ y_k &= x_k + \xi_k, \end{aligned} \tag{6}$$

where $\eta_k$ is the input system white noise (uncertainty) mentioned above. Consequently, in this case, $\mathbf{h} = 1$ and the error covariance matrix $\mathbf{P}_{k+f|k}$ is actually a simple scalar variance and the $\hat{y}_{k+f|k} = \hat{x}_{k+f|k}$ because $\xi_k$ is assumed to be zero mean, white noise.

In the forecasting system optimization, which is carried out separately to the model estimation, the gain and variance hyper-parameters $q_k$, $\sigma_0^2$ and$\sigma_1^2$ are estimated by minimising a modified cost function based on the sum of the squares of the $f$-step-ahead forecasting errors, with an additional term ensuring that 95% of the observation points are situated within the lower and upper 95% confidence bounds: i.e.,

$$\begin{aligned} J &= \sum_{k=1}^{T-f} \frac{(\hat{y}_{k+f|k} - y_{k+f})^2}{\mathrm{cov}(y_k)} + \left( \frac{\sum i}{T-f} - 0.95 \right)^2, \\ i &= \begin{cases} 1 & y_{k+f} \in [\hat{y}_{l,k+f|k}, \hat{y}_{u,k+f|k}] \\ 0 & y_{k+f} \notin [\hat{y}_{l,k+f|k}, \hat{y}_{u,k+f|k}] \end{cases}, \quad k = 1, \ldots, T-f, \end{aligned} \tag{7}$$

where $\hat{y}_{l,k+f|k}$ and $\hat{y}_{u,k+f|k}$ denote the estimates of the lower and upper 95% confidence bounds, respectively: i.e.,

$$\begin{aligned} \hat{y}_{l,k+f|k} &= \hat{y}_{k+f|k} - 1.96 \cdot \sqrt{\mathrm{var}(\hat{y}_{k+f|k})}, \\ \hat{y}_{u,k+f|k} &= \hat{y}_{k+f|k} + 1.96 \cdot \sqrt{\mathrm{var}(\hat{y}_{k+f|k})} \end{aligned} \tag{8}$$

and $\hat{y}_{k+f|k} = \hat{p}_k \cdot \hat{x}_{k+f|k}$ denotes the updated $f$-step-ahead forecast, while $T$ denotes the time horizon of the estimation (calibration) stage.

### 2.2. Comments

1. Any model needs to be validated in some manner if it is to be used with confidence in any particular application. In the case study described in the next Section 3, we utilize the standard approach of evaluating the complete system, as obtained in the previous Section 2.1, on separate validation data sets not used in the system identification, estimation and design stages.

2. In the event that a downstream site fails, for some reason, so that data are not available from this site, the model can continue to estimate the propagation of the flood wave downstream (although clearly without the improvements in forecasts allowed by the on-line updating). The forecast lead time may also be increased by combining the modules for the upstream reaches, where forecasting over a period longer than the natural time delay is required, as in [7]. Note that this procedure requires derivation of new gain and variance hyperparameters for the changed forecast lead-times.
3. It is interesting to note that, under the assumption that the *f*-step-ahead prediction error is dominated by the first two moments of the associated probability distribution, this formulation allows the uncertainty in the representation of the nonlinear transform, the model parameters and observation errors to be accounted for in a simple manner. It is easily implemented, since the statistical assumptions result in the 95% confidence bounds being approximately equal to twice the standard deviation obtained from (4) at each time step. In the case where multistage forecasts are applied, the uncertainty of the forecasts in the upstream reach can be propagated into the downstream forecasts [40].
4. If forecasts are required at multiple lead times, including the use of outputs from upstream predictions, the variance parameters (4) should be re-optimized, together with the gain hyperparameter for the forecast inputs, at each forecast lead time. The errors related to model structure, such as the STF model parameters, play a different role in the overall uncertainty of the predictions than the input uncertainty, which includes the nonlinear transformation of water levels. As the weights for the nonlinear SDP parameterisation are optimized simultaneously with estimation of the STF function parameters, the uncertainty related to these model parameters is quite small when compared with the uncertainty due to the observational errors (recall from our previous comments that, in relation to the model (2), these are associated with any stochastic inputs affecting the model, including measurement noise, unmeasured inputs, and errors arising from uncertainties associated with the model structure).
5. Finally, it should be noted that the general procedure described above is very flexible. For instance, it has clear relevance for flow routing applications. Also, it can be used in a hybrid computational context, with uniform or irregularly sampled data. In the case of uniform discrete time samples, the reach model is identified and estimated in continuous-time, differential equation terms using the SDP algorithm, in combination with the Refined Instrumental Variable algorithm for Continuous-time systems (RIVC: see [41] and references therein). In this case, the Kalman Filter forecasting algorithm is formulated in similar hybrid, continuous–discrete time terms, with a continuous-time prediction step followed by discrete-time updating of these predictions. In the irregularly sampled situation, it is straightforward to update both the RIVC algorithm and the Kalman Filter to allow for the non-uniform sampling.

## 3. Case study: Forecasting on River Severn between Welsh Bridge and Bewdley

This section describes the application of the methodology described in the previous Section to an 80 km long reach of the River Severn in the UK between Welsh Bridge, near Shrewsbury, and Bewdley, as shown in Fig. 1. The observed hourly water levels at Welsh Bridge, Buildwas, Bridgnorth and Bewdley were obtained from the UK Environment Agency, Midlands Region. This part of the River Severn was chosen as a development of the previous work on the on-line updated forecasting system [7]. The maximum water levels for the flood events in 1998, 2000 and 2002 at each location are given in Table 1.

**Table 1**
Magnitude of flood peaks (water levels (m)) at Welsh Bridge, Buildwas, Bridgnorth and Bewdley for 3 flood events in 1998, 2000 and 2002

| Station/Peak height (m) | 1998 | 2000 | 2002 |
|---|---|---|---|
| Welsh Bridge | 4.86 | 5.24 | 4.67 |
| Buildwas | 6.13 | 7.03 | 5.87 |
| Bridgnorth | 4.90 | 5.26 | 4.62 |
| Bewdley | 4.98 | 5.56 | 4.63 |

### 3.1. Design and application of the SDP–STF based flood forecasting system for the River Severn reach

All of the identification and estimation algorithms used in the model calibration stage of the analysis, as outlined in the previous Section, are available in the CAPTAIN Toolbox[2] for Matlab™ (see e.g. [42]) and were used in this example. However, the adaptive forecasting system, based on these models, is programmed as a special Matlab program (m-file), wherever necessary calling on the CAPTAIN algorithms, and it is not available so far as a routine in CAPTAIN.

In the first step of the model calibration, the application of SDP estimation to water levels at the three reaches for a period in 1998–2000, including the flood peaks of Table 1, results in the non-parametric (graphical) estimates of the nonlinear function for Welsh Bridge–Buildwas shown in Fig. 3, with the dotted lines indicating the estimated 95% confidence bounds and thick black line indicating the finally optimized radial basis function estimate of the SDP relation. The shape of the nonlinear gain depends on the relative differences in the geometry of the river cross-section areas at the downstream and upstream locations. In this particular case, its values are greater than one indicating that water levels at Buildwas are higher than those at Welsh Bridge. The bank-full levels are equal to about 2.8 m and 5.0 m at Welsh Bridge and Buildwas, respectively. Therefore, the range of water levels near the peak of the gain (3 m at Welsh Bridge) corresponds to the bank-full level at Buildwas. The gain decreases with further increase of water lev-

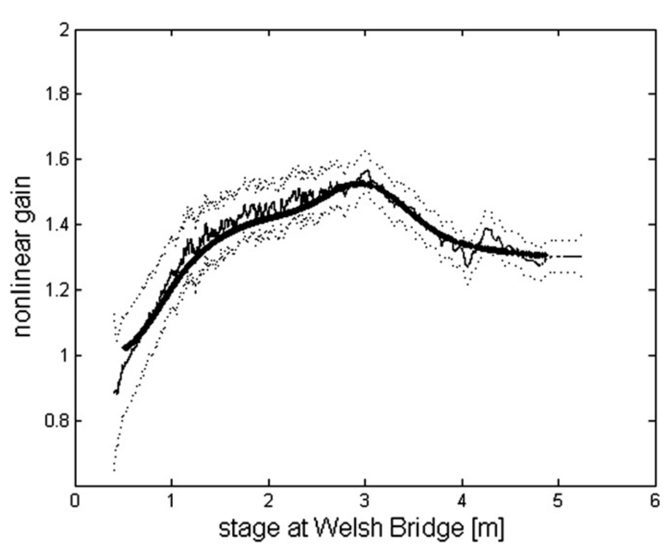


**Fig. 3.** SDP function for water level relationship between Welsh Bridge and Buildwas with the dotted lines indicating the estimated 95% confidence bounds and thick black line showing the finally optimized SDP relation using radial basis functions.

els at Welsh Bridge indicating engagement of the floodplains near Buildwas in flow routing.

After the initial non-parametric identification of the STF model structure, the nonlinear SDP gains for each reach are parameterised using a radial basis function (a weighted sum of Gaussian-shaped functions: see e.g. [38]). In this example, it is found that a radial basis function with ten elements is sufficient for all the reach models. The radial basis function parameters are optimized simultaneously with the STF parameters in order to obtain the best forecasting model for the water levels. The parameters are estimated on the basis of the 1998–2000 data and validated on the year 2002 event. The STF $[n \quad m \quad \delta]$ model structure is re-identified at this stage but, as pointed out previously, the simplest, first order STF model $[1 \quad 1 \quad \delta]$ is found to be adequate in all cases.

As a typical example of the above procedure, let us consider the STF model identification and estimation for the Welsh Bridge to Buildwas reach. Here, the RIV algorithm in the Captain Toolbox is used to first identify the STF model structure and then estimate the associated parameters. The best identified model structure is $[1 \quad 1 \quad 8]$ and has the form (combining the two equations of the general model (2)):

$$y_k = \frac{\hat{b}_0}{1 + \hat{a}_1 z^{-1}} \tilde{u}_{k-8} + \xi_k, \tag{9}$$

where the estimated parameters are $\hat{b}_0 = 0.4685(0.0192)$ and $\hat{a}_1 = -0.5245(0.0194)$, with the figures in parentheses denoting the estimated standard errors on the estimates. Here, $\tilde{u}_k = g(u_k) \cdot u_k$ represents the water level at Welsh Bridge transformed by the estimated SDP nonlinearity; $y_k$ is the water level measured at Welsh Bridge; the identified model advective delay is 8 h; and $\xi_k$ denotes the modelling error and noise effects. The same approach is applied to the other reaches and the estimated STF model parameters for all the reaches are listed in Table 2, together with their estimated standard errors. The residence and travel time parameters derived from the STF model parameters that, together with the advective pure time delay, define the dynamics of the reach, are also shown. Following the notation from Eq. (2), all the $[1 \quad 1 \quad \delta]$ STF models have one parameter in the denominator ($a_1$) and one in the numerator ($b_0$).

At the forecasting stage of the procedure outlined in the previous Section, the STF model is re-formulated in a state space form (see [27,7]) and the hyper-parameters for the noise variance ratio parameters are estimated by maximum likelihood, using a Kalman filter, together with the prediction error decomposition approach [43], to derive the likelihood function. Note that either an Auto Regressive (AR) or an Auto Regressive, Moving Average (ARMA) model for the noise $\xi_k$ may also be included in the state space formulation at this stage. However, we found that this only improved the one-step-ahead forecast performance and did not give a sufficient improvement for the longer lead times to justify the increase in model complexity. Nevertheless, we believe that this aspect of the modelling should receive further investigation, since the lack of significant improvement probably derives from the inadequacy of the AR model as a representation of the noise process: a more sophisticated stochastic model, possibly integrating the heteroscedastic characteristics, should result in at least some further improvement in the forecasting performance.

Finally, in the last step of the forecasting system design procedure, the on-line adaptive gain and variance hyper-parameters are optimized using the criterion function in Eq. (7). This is an important step in the design because it is a crucial stage in tuning the system to achieve good on-line, real-time forecasting performance, which is the main objective of the study in this case. However, the performance in this regard can only be evaluated objectively during the final validation stage of the design procedure.

The validation results for all three gauging stations, based on the 2002 flood events, are summarised in Table 3, where the forecasting performance is reported in terms of two statistical measures: (i) the *f*-step-ahead coefficient of determination,

$$R_f^2 = 1 - \frac{\sigma_f^2}{\sigma_y^2}, \tag{10}$$

where $\sigma_f^2$ and $\sigma_y^2$ denote the variances of *f*-step ahead forecast error and observed water level, respectively; and (ii) the root mean square error (RMSE) defined as follows:

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{k=1}^{T-f} (y_{k+f} - \hat{y}_{k+f|k})^2}{T-f}}, \tag{11}$$

where $\hat{y}_{k+f|k} = \hat{x}_{k+f|k}$ denotes *f*-step ahead forecast, evaluated over the time horizon $T - f$, where $T$ is the total sample length. Due to the fact that a nonlinear SDP transformation is applied to the upstream water levels at each reach, the forecasts can be extended, as in [7], by using the forecast upstream water levels instead of measurements. The lead time can be increased still further by the use of rainfall–runoff models in the upstream catchment areas and tributaries (e.g. upstream of Abermule, see [7]). Obviously, however, the quality of the forecasts will decrease with an increase in the lead time. The last three rows of Table 3 show the various statistical criteria obtained after combining the individual reach forecasts in this manner: we see that the forecast at Bewdley may be

**Table 2**
STF model parameters for River Severn reaches between Welsh Bridge and Bewdley

| Location | $\hat{a}_1$ | $\hat{b}_0$ | $\hat{\delta}$ (h) | Residence time (h) | Travel time (h) |
|---|---|---|---|---|---|
| Welsh Bridge–Buildwas | 0.5245 (0.0192) | 0.4685 (0.0194) | 8 | 1.55 | 9.55 |
| Buildwas–Bridgnorth | 0.3710 (0.0282) | 0.6544 (0.0293) | 2 | 1.01 | 3.01 |
| Bridgnorth–Bewdley | 0.4609 (0.0220) | 0.6478 (0.0264) | 4 | 1.29 | 5.29 |

**Table 3**
$R_f^2$, *RMSE* and absolute error for the maximum peak wave obtained in the validation stage of the on-line forecasting (2002 flood event)

| River Severn reach | Lead time (h) | $R_f^2$ (%) | RMSE (m) | Abs error for the peak (m) |
|---|---|---|---|---|
| Welsh Bridge–Buildwas | 8 | 99.89 | 0.076 | 0.097 |
| Buildwas–Bridgnorth | 2 | 99.93 | 0.029 | 0.006 |
| Bridgnorth–Bewdley | 4 | 99.80 | 0.066 | 0.019 |
| Buildwas–Bewdley | 6 | 99.64 | 0.085 | 0.061 |
| Welsh Bridge–Bridgnorth | 10 | 99.26 | 0.096 | 0.085 |
| Welsh Bridge–Bewdley | 14 | 98.83 | 0.139 | 0.169 |

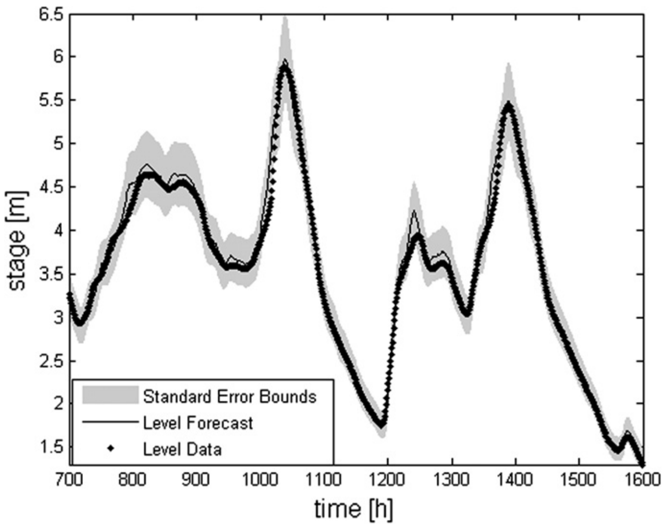


**Fig. 4.** 2002 flood event (30.01.2002–9.03.2002): validation of on-line adaptive 8-h-ahead on-line adaptive forecast for Buildwas (thin black line); the shaded areas denote the 95% confidence bounds; the black dotted line denotes the observations. 99.65% of the output variation explained by the forecasts.

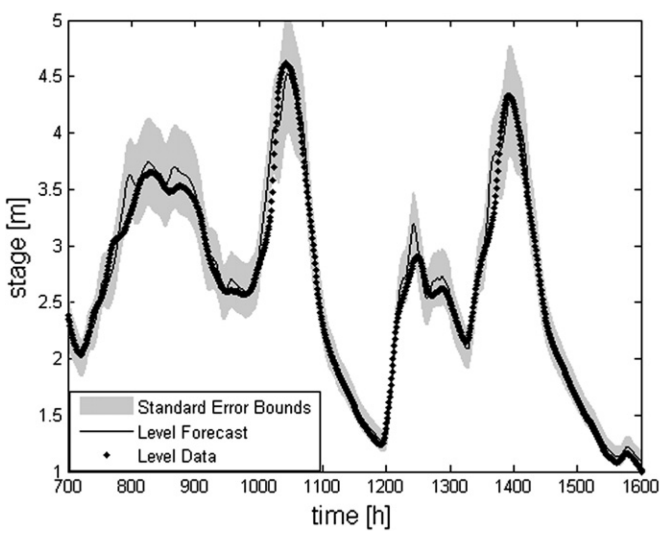


**Fig. 5.** 2002 flood event (30.01.2002–9.03.2002): validation of 10-hour-ahead on-line adaptive forecast for Bridgnorth (thin black line); the shaded areas denote the 95% confidence bounds; the black dotted line denotes the observations. 99.26% of the output variation explained by the forecasts.

extended to a maximum of 14 h when the 10-h-ahead forecast for Bridgnorth is used.

The statistical criteria (9) and (10) are evaluated for the whole 2500 h validation period in 2002. This period includes the two significant flood peaks shown in Figs. 4–6 below. The on-line, 8-h-ahead forecast for the validation stage on the 2002 flood event for the Welsh Bridge to Buildwas reach is shown in Fig. 4. This model explains 99.65% of the output water level variations measured at Buildwas.

Fig. 5 shows the 10-h-ahead forecast obtained using the model (2) for the Buildwas to Bridgnorth reach over the 2002 flood event. It is derived as a result of combining the Buildwas 8-h-ahead-forecast with the Bridgnorth 2-h-ahead forecast and it explains 99.26% of the output water level variations at Bridgnorth. Finally Fig. 6 shows the 14-h-ahead forecasts obtained for the same 2002 event at Bewdley, resulting from the combination of the forecast for

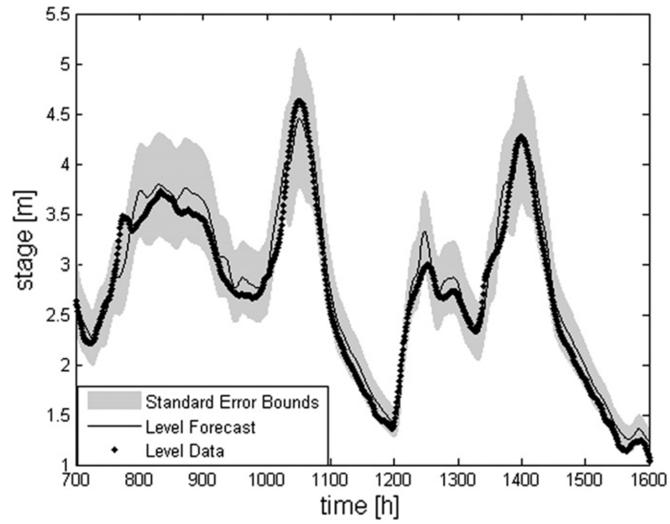


**Fig. 6.** 2002 flood event (30.01.2002–9.03.2002): validation of on-line, adaptive 14-hour-ahead forecast for Bewdley (thin black line); the shaded areas denote the 95% confidence bounds; the black dotted line denotes the observations. 98.83% of the output variation explained by the forecasts.

Bridgnorth, from Fig. 5, with the on-line 4-h-ahead forecast for the Bridgnorth–Bewdley reach: it explains 98.83% of the output water level variations at Bewdley.

Note that the changing width of the 95% confidence bounds over time in Figs. 4–6, is a consequence of the state-dependent heteroscedastic modelling of the observational noise process and it shows how, as expected, the maximum uncertainty in the multi-step-ahead forecasts is associated with the maximum flows.

In order to apply the above methodology operationally, different model set-ups might be required for different lead times. For example, in the River Severn, a greater than 30-h-ahead forecast would require the incorporation of rainfall–runoff models, water level routing and the addition of tributary inflows (as demonstrated in [7,28]). A 12-h-ahead forecast might require, as input, the water level information from a single station tens of kilometres upstream; while, for increased accuracy at shorter lead times during a major event, a 6-h-ahead forecast might only require information from the station immediately upstream.

### *3.2. Comparison with an alternative system*

The results presented here compare favourably with the results of the on-line inundation updating experiment for the River Severn reach using a 1-D flow routing model (ISIS, HR Wallingford) reported in [22]. The distributed ISIS model was applied to the River Severn reach between Montford and Buildwas. The model was applied within the Generalised Likelihood Uncertainty Estimation (GLUE) framework in order to estimate the confidence bounds for the forecasts. The entire estimation procedure required about 12 h of computer time, not to mention the effort of applying the updating routine for the inundations, involving the Kalman filter-based water level forecasting similar to the one presented here. The experience gained from this work was the major driving force for the simplification of the data assimilation routine described in the present paper.

The ISIS model requires three roughness coefficients for each cross-section: one for the channel and two for the floodplains (though only two values, one for the channel and one for both floodplains, are routinely used for the entire modelled reach). Our experience shows that the model could be reliably calibrated only for the cross-sections where the water level observations were available (Welsh Bridge at Shrewsbury). Elsewhere, comparisons with inundation extent estimated from air photography and SAR data suggested that there were some cross-sections where the hydraulic model could not provide good simulations, despite a very wide sampling of parameter values, which could also be due to the uncertainty in determining the edge in the SAR image [44]. The forecast lead – time was equal to the natural time delay required by the flood wave peak to travel from Montford to Buildwas (Fig. 1). Taking into account the computer time required to obtain these forecasts, together with uncertainty estimates for the hydraulic model, the procedure proposed in the present paper, using of the order of only seconds of computer time, is much more efficient. In the case when distributed forecasts are required by the user, a distributed model simulator allowing for the interpolation between the observation sites seems to be a more suitable solution. Work is currently being carried out at Lancaster University on this type of modelling tool as an alternative to the distributed modelling approach for on-line flood forecasting.

## 4. Discussion and conclusions

This paper describes an approach to on-line, real-time flood routing and forecasting which involves a nonlinear, stochastic model of the flood routing process. The nonlinear model for each defined reach, which is identified from the river level data at adjacent locations along the river, consists of a state-dependent parameter nonlinearity in series with a linear transfer function, with the latter interpreted as one or more storage elements. The novelty of this model lies in the nonlinear transformation of the upstream input water levels, which effectively separates the processes of lateral wave transformation from the longitudinal dispersion. We are aware that many other linear and nonlinear representations of the flow routing process are possible. However, the statistical approach proposed here is inductive, with the model identified and estimated directly from the measured data, so that it does not rely on the efficacy of any *a priori* assumed model form. Moreover, it is simple yet inherently stochastic, so allowing for fast, reliable on-line data assimilation and forecast updating techniques to be employed.

The proposed methodology has been evaluated on three reaches of the River Severn in the UK (Fig. 1) from Welsh Bridge down to Bewdley. The overall model is stochastic so that, in addition to the estimated and forecast mean values, the forecasts have associated estimates of the confidence bounds, thus providing information about the uncertainty involved in the multi-step-ahead forecast of the river level. The proposed decomposition of the model into the nonlinear transformation and longitudinal dispersion works well when the correlation between the water levels at two neighbouring cross sections is high, as is often the case in practice. Of course, as in the case of all models estimated on the basis of real data, the forecasts will be sensitive to changes in gauging station and flood plain geometry, relative to the period of calibration, so that the nonlinear state dependent parameter transformation might need to be re-adjusted after significant changes. In the present study, however, there is no evidence in any significant change between the 1998 and 2000 floods, used for the model/forecasting system estimation (calibration), and the 2002 floods, used to validate the model and its associated forecasting system. Moreover, if re-estimation proved necessary, it is a straightforward exercise with such a simple model and could be accomplished on-line. In addition, the inherent adaptive adjustment of the forecasts during an event means that some allowance for smaller changes in the river dynamics is included in the forecasting system.

The approach to model estimation and forecasting described here requires a good, reliable water level monitoring system along the river, with special emphasis on the high flood-risk areas. Nevertheless, its application is much less data-intensive than the full application of a distributed flood routing model, with an associated computing time of seconds rather than hours. Moreover, the forecasts are supplied with 95% confidence limits, taking into account uncertainty in the modelling. The methodology opens up some interesting possibilities, including the installation of networked depth sensors to obtain specific local forecasts for sites at risk; self-calibration of the SDP/STF models for such sites; and the integrated identification of models for a complete network. These possibilities are the subject of continuing research. Some work already has been done towards further simplification of the forecasting procedure. Namely, a delay varying with the water level heights was introduced instead of STF filtering [45]. That approach was tested on the same River Severn reach as the application presented here and the results are comparable but give larger forecast errors due to a lack of smoothing effects of filtering.

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