Some aspects of the HARMONIE limited-area model

M. Hortal

HIRLAM Project Leader for Dynamics
AEMET (Spain)

Abstract

Some improvements in the HARMONIE limited-area model, being developed by the HIRLAM consortium, in collaboration with ALADIN and ECMWF, are described in this presentation. In the first place, the adaptation to the non-hydrostatic version of the model, of the finite-element procedure in the vertical which was implemented by Untch & Hortal on the hydrostatic version. In the second place, the procedure to apply the boundary conditions (developed by Radnoti) and the periodization of the fields are being modified in order to allow a large extension zone, which eliminates some problems seen in the data assimilation using HARMONIE. This involves the use of Boyd's biperiodization and Davies relaxation applied in spectral space.

1. Introduction

The HARMONIE model (**H**irlam and **A**ladin **R**esearch **M**odel **O**n **N**on-hydrostatic-forecast **I**nside **E**urope) is part of the "galaxy of models" referred to in Pierre Bénard's lecture (this volume), which include the IFS (ECMWF), ARPEGE (Météo-France), ALADIN (Aladin consortium), ALARO (the version adapted to the "grey zone" developed mainly by the LACE consortium), AROME (Meso-NH group) and HARMONIE (the version of HIRLAM).

All of these models use the same code for the dynamical part, which is a semi-Lagrangian semi-implicit two-time-level time stepping scheme based on a spectral representation in the horizontal.

The main differential characteristics being:

- IFS is a uniform-resolution version on the globe, using a reduced Gaussian grid and spherical harmonics as the horizontal spectral basis.

- ARPEGE is a global version using a stretched Gaussian grid which allows a higher resolution on a selected part of the globe, the resolution decreasing as we go to the opposite part of the globe.

- ALADIN is a version on a limited area, on a plane projection of the sphere and with the option of non-hydrostatic dynamics.

- ALARO uses the same dynamics (hydrostatic/non-hydrostatic) as ALADIN but the physical parameterizations are targeted to the "grey zone", the resolutions at which the deep convection starts to be represented by the resolved dynamics.

- AROME uses the non-hydrostatic dynamics and physical parameterizations developed for the resolutions in which the deep convection is quite well resolved by the dynamics.

- HARMONIE is the limited-area version used by the HIRLAM consortium and encompasses all the dynamics options and the physical parameterizations mentioned above, plus a system of scripts and namelists which allows experiments to be submitted under a scheduler (mini-SMS).

2. Finite element discretization in the vertical

Untch & Hortal (2004) describe the application of the finite-element technique to compute the only vertical operator needed in the hydrostatic semi-Lagrangian version of the IFS model, the integral operator. The technique adopted is not a representation in finite-element space of the fields of the model, but the equivalent to the spectral transform method, where most of the computations are done in grid-point representation, using a non-staggered grid, and the transformation to finite-element space is only performed in order to compute the vertical integrals.

The finite-element technique was applicable to the case of the semi-Lagrangian hydrostatic version because only the integral operator was used, and the constraints to eliminate all the fields but one from the linear model of the semi-implicit scheme, to arrive at a single Helmholtz equation for the divergence, were fulfilled by the integral operator computed with the finite-element technique.

This is not the case in the non-hydrostatic version, used at the resolution of 2.5 km in the HARMONIE model. Both integral and derivative operators are needed. These operators, as computed with the finite-element technique, are not the exact inverse of each other and furthermore, constraint C1 (Bubnova et al 1995) is not fulfilled by the integral operator computed by means of finite-elements.

Two different avenues have been attempted in order to solve this obstacle:

- The first one is to change the set of equations or the vertical coordinate in order to eliminate the need of the use of the vertical integral. This gave the possibility of arriving at a single Helmholtz equation in the vertical divergence, but the application in the vertical of any operator (either finite-differences or finite-elements) of order greater than 4 led to instability.

- The second one has been developed by the LACE group (Vivoda & Smolíková 2013) and is based on not applying constraint C1, therefore arriving, not at a single Helmholtz equation but to a coupled system of two equations in the horizontal and the vertical divergences.

Let us first describe the way to use the finite-element technique to compute a vertical operator, e.g. the vertical derivative.

We want to discretize, using the finite-element technique, the derivative operator:

$$F(\eta) = \frac{df(\eta)}{d\eta} \tag{1}$$

We approximate both the unknown function F and the initial function f as linear combinations of basis functions of a Hilbert space covering our domain of interest ($0 \leq \eta \leq 1$).

$$F(\eta) \approx \sum_{i=1}^{M} F_i E^i(\eta) \quad ; \quad f(\eta) \approx \sum_{i=1}^{N} f_i e^i(\eta) \tag{2}$$

The basis functions are not necessarily the same for the function and for its derivative and they can include boundary conditions. For example the basis set chosen for the representation of the vertical integral from the top of the atmosphere in Untch & Hortal were all of them null at the top of the atmosphere (η=0).

The original identity (1) then becomes the approximation:

$$\sum_{i=1}^{M} F_i E^i(\eta) \approx \sum_{j=1}^{N} f_j \frac{de^j(\eta)}{d\eta} \tag{3}$$

Applying now the Galerkin procedure, we prescribe the truncation error to be orthogonal to the space spanned by a certain set of basis functions called the test functions $T_k(\eta)$.

$$\sum_{i=1}^{M} F_i \int_0^1 E^i(\eta) T_k(\eta) d\eta = \sum_{j=1}^{N} f_j \int_0^1 \frac{de^j(\eta)}{d\eta} T_k(\eta) d\eta \tag{4}$$

This can be expressed in matrix form as:

$$\widetilde{F}\mathbf{A} = \widetilde{f}\mathbf{B}$$

Where matrix $\mathbf{A} => (A_k^i) \equiv \int_0^1 E^i(\eta) T_k(\eta) d\eta$ is called the mass matrix and matrix $\mathbf{B} => (B_k^j) \equiv \int_0^1 \frac{de^j(\eta)}{d\eta} T_k(\eta) d\eta$ is the derivative matrix

This can be solved if matrix **A** is invertible (non-singular) as

$$\widetilde{F} = \widetilde{f}\mathbf{B}\mathbf{A}^{-1}$$

The vector $\widetilde{f}$ is the set of components of function $f(\eta)$ approximated as linear combination of the basis functions $e^j(\eta)$, but the information we have about this function consists on its values at the full levels of the model $f(\eta_l)$, therefore we have to compute the components from the point values (projecting on the finite-element space):

$$f(\eta_l) = \sum_{j=1}^{N} f_j e^j(\eta_l) => \hat{f} = \widetilde{f}\mathbf{P} => \widetilde{f} = \hat{f}\mathbf{P}^{-1}$$

Similarly, once we know the components $\widetilde{F}$ we can compute the values of the function represented by these components, at the full levels of the model:

$$F(\eta_l) = \sum_{j=1}^{M} F_j E^j(\eta_l) => \hat{F} = \widetilde{F}\mathbf{S}$$ where matrix **S** is the inverse projection operator corresponding to the finite-element space defined by the basis functions *E*.

Finally, we have:

$$\hat{F} = \tilde{F}\mathbf{S} = \tilde{f}\mathbf{B}\mathbf{A}^{-1}\mathbf{S} = \hat{f}\mathbf{P}^{-1}\mathbf{B}\mathbf{A}^{-1}\mathbf{S} \equiv \hat{f}\mathbf{D}$$

Where **D** is a matrix which, applied on the vector of values of a function *f* gives the vector of values of its derivative *F*, on the same points as the original function is given.

In identical way we can compute the matrix **C** for the integral of a given function.

The first problem encountered is that matrices **D** and **C** are not the inverse of eachother and furthermore, constraint C1 of Bubnova et al:

$\mathbf{A_1} \equiv \mathbf{G}^*\mathbf{S}^* - \mathbf{S}^* - \mathbf{G}^* + \mathbf{N}^* = 0$ is not fulfilled. Here the operators **G***, **S*** and **N*** are defined as:

$$\left(\mathbf{G}^*\hat{\psi}\right)_l \equiv \int_{\eta_l}^{1} \frac{m^*}{\pi^*}\psi d\eta \quad \left(\mathbf{S}^*\hat{\psi}\right)_l \equiv \frac{1}{\pi_l^*}\int_0^{\eta_l} m^*\psi d\eta \quad \text{and} \quad \left(\mathbf{N}^*\hat{\psi}\right)_l \equiv \left(\mathbf{S}^*\hat{\psi}\right)_{L+1}$$

L is the number of full levels in the model L+1 means the surface of the model (where η=1), and subindex *l* indicates the value at model level *l*.

π^* is the reference hydrostatic pressure for the linearized system, and m^* its derivative with respect to the vertical hybrid coordinate η , *T** (see later) is the reference temperature for the linearization.

The solution implemented by Vivoda & Smolikova is to stop the elimination procedure in the semi-implicit scheme at the point where constraint C1 is needed to proceed. They therefore do not arrive at a single Helmholtz equation but to a system of two coupled equations in both the vertical and the horizontal divergences:

$$\begin{pmatrix} \mathbf{E} & -\mathbf{F} \\ -\mathbf{B} & \mathbf{A}+\mathbf{C} \end{pmatrix} \begin{pmatrix} d \\ D \end{pmatrix} = \begin{pmatrix} d^\bullet \\ D^\bullet \end{pmatrix} \tag{5}$$

where $d^\bullet$ and $D^\bullet$ are the r.h.s. of the linearized equations for the vertical and the horizontal divergences, when the elimination procedure is stopped.

Matrices **E, F, B, A** and **C** are:

$$\mathbf{E} = \left(1 - \Delta t^2 c^2 \frac{\mathbf{L}^*}{rH^2}\right), \quad \mathbf{F} = \Delta t^2 \frac{\mathbf{L}^*}{rH^2}\left(-RT^*\mathbf{S}^* + c^2\right), \quad \mathbf{B} = \Delta t^2 \nabla^2\left(-RT^*\mathbf{G}^* + c^2\right),$$

$$\mathbf{A} = \left(\mathbf{I} + \Delta t^2 c^2 \nabla^2\right) \quad \text{and} \quad \mathbf{C} = \Delta t^2 \nabla^2 RT^* \mathbf{A_1}$$

Where operator $\mathbf{L}^*$ is called vertical Laplacian and is defined as

$$\left(\mathbf{L}^*\hat{\psi}\right)_l = \left(\frac{1}{m^*}\frac{\partial}{\partial\eta}\left(\frac{\pi^{*2}}{m^*}\right)\frac{\partial\psi}{\partial\eta} + \left(\frac{\pi^*}{m^*}\right)^2 \frac{\partial^2\psi}{\partial\eta^2}\right)_l \tag{6}$$

The system of equations (5), unlike the Helmholtz system in the finite differences version, which is of dimension the number of levels, has a dimension of double the number of levels. There are two options to solve this system

- Solve directly the system of dimension 2*L. This requires a large amount of memory and therefore the other possibility is favoured:

- Do an iterative procedure solving a system of dimension L at each iteration.

Both of them are described in Vivida and Smolikova and will not be given here.

The result appears to be a stable scheme and, if cubic polynomials are used as the basis for the computation of the vertical operators, eighth order of accuracy is achieved far from the upper and lower boundaries.

3. Application of boundary conditions and biperiodization in spectral space

The HARMONIE limited area model solves the same system of equations as the global IFS version but, instead of using a spherical harmonics basis for the horizontal representation of fields in spherical coordinates, it uses a bi-Fourier representation on a plane projection using Cartesian coordinates.

For a representation using Fourier basis functions, the fields need to be periodic in both the x and the y directions. As in a limited area domain this condition is not fulfilled, an extension zone is added. After the computations in grid-point space, this extension zone is filled with artificial data in such a way that the values of a field at the right hand edge of the domain join smoothly with its values at the left hand edge, and also the ones at the upper edge join smoothly the ones at the lower edge.

The procedure, called biperiodization, is performed by means of cubic splines along each row (direction x) and each column (direction y) of values.

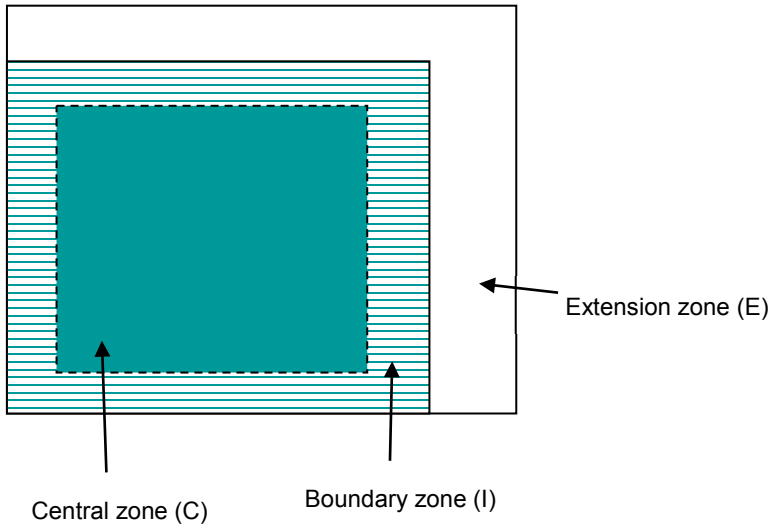
Two problems appear due to this procedure:

- The grid-points on the extension zone are a part of the normal computations in grid-point space, adding cost to the grid-point space computations, but the results are not used: they are overwritten by the biperiodization procedure.

- In data assimilation, an observation close to the boundary of the domain can influence the analysis at the opposite boundary, which is unphysical. Increasing the size of the extension zone alleviates this problem but worsens the first one.

The solution to this dilemma is to eliminate the extension zone from the grid-point representation in the model.

The distribution of the domains used in the model are as follows:



Computations are needed on the two zones C and I, then boundary conditions should be applied on zone I using information coming from a "nesting model" and finally a biperiodization introducing artificial data on zone E is needed before going to spectral space.

The procedure is described in Radnoti (1995) and is as follows:

The application of the semi-implicit scheme for the solution of a certain field can be written as:

$$(I-\Delta t\mathscr{L}\ )\Psi_{t+\Delta t}=\Psi_{t+\Delta t(\exp)}+\Delta t\mathscr{L}\ \left(\Psi_{t-\Delta t}-2\Psi_{t}\right)$$

Where the first term of the r.h.s. is the solution of the equation for the field Ψ, at time step t+ Δt, using an explicit time stepping scheme. $\mathscr{L}$ is the linear operator corresponding to the linearization used for the semi-implicit scheme.

Then the Davies-Kallberg relaxation procedure can be expressed as:

$$\Psi^{C}=\left(1-\alpha\right)\cdot\Psi^{l}+\alpha\cdot\Psi^{LS} \tag{7}$$

Where Ψ^{l} is the value of the field at a certain grid-point, computed either with the explicit or the semi-implicit time-stepping scheme, Ψ^{LS} is the value coming from a nesting model (larger scale), at the appropriate moment in time, and α is a relaxation function which changes smoothly from zero at the edge of the C zone to one at the edge of the I zone. Expression (7) can include the biperiodization, if the field Ψ^{LS} includes a smooth transition from every edge of the I zone to the opposite edge. This transition can be computed with only values of the nesting model, as the information on the edge of zone I comes exclusively from this nesting model, and it might be done by either splines or the Boyd procedure.

Boyd's procedure consists of taking, from the nesting model, information over an area symmetric to the extension zone with respect to the C+I zone and combining the opposite sections, multiplied by transition functions changing smoothly from 0 to 1. (for more details, see Boyd 2005).

The implementation by Radnoti then reads:

$$\left(I-\Delta t\mathscr{L}\right)\Psi_{t+\Delta t}=\left(1-\alpha\right)\Psi^{I}+\alpha\left(I-\Delta t\mathscr{L}\right)\Psi^{LS}_{t+\Delta t} \tag{8}$$

The fields coming from the nesting model Ψ^{LS} , already biperiodized using either splines or Boyd's procedure, are interpolated linearly in time to the end of the present time step and multiplied by the Helmholtz operator *(I-Δt* $\mathscr{L}$ *)* and by the Davies relaxation function *α.* This is added to the right-hand-side of the corresponding equation, multiplied by *(1-α)* and the result transformed to spectral space, where the Helmholtz operator is inverted. The result is a field identical to the nesting information over the extension zone and identical to the solution of the semi-implicit scheme over the inner zone C. The horizontal derivatives of this field can be computed in spectral space and transformed to grid-point space. They are therefore completely consistent with the fields themselves.

In order to perform the above computations, the extension zone should be included in the grid-point representation.

The proposed modification, which is at the moment being included in the HARMONIE model is to compute $\alpha\left(I-\Delta t\mathscr{L}\right)\Psi^{LS}$ at the coupling times (this is done before the forecast itself starts, when the initial and boundary conditions are prepared for the forecast run, and uses the present configuration of the model, including the extension zone in grid-point space), instead of the Ψ^{LS} , and keep this information in spectral space. These fields are written on an extra boundary file.

At the time of running the forecast, the extra boundary file is read and the spectral fields corresponding to the coupling times are linearly interpolated in time to the moment of the end of the time step. As the function *α* and the operator $\left(I-\Delta t\mathscr{L}\right)$ are independent on time and the spectral transforms are linear operators, the result is identical to first interpolate Ψ^{LS} in time and then apply the operator and multiply by *α*.

The forecast then proceeds as follows: in grid-point space (where the extension zone is not included), the values of the Ψ^{I} of equation (8) are computed and multiplied by *(1-α)*. These fields are then zero at the border of the C+I area. During the spectral transform, all the points belonging to a row (the same value of coordinate y for all of them) are communicated to one processor (transposition to prepare the first Fourier transform), in order to allow a Fourier transform. These values are zero at both the left-hand side and the right-hand side of C+I and therefore can be made periodic over the C+I+E dimension simply padding with zeros. The Fourier components therefore correspond to C+I+E area. Then another transposition is done so that the same wavenumber m corresponding to all the values of the coordinate y are present in the same processor. Again the values at the ends of the interval inside C+I are zeros and then we can pad with zeros to the C+I+E length and perform the second Fourier transform (in direction y). Finally in spectral space, we add the time-interpolated value coming from the nesting model. The result will not be bit-identical to the present procedure because we change the order of some of the operations (the time interpolation is done after the spectral transforms) but it would have been identical in the continuous case.

Using this procedure of elimination of the extension zone from the grid-point representation of fields, we can increase the width of the extension zone without any cost in the grid-point computations (where most of the cost of the model resides) and eliminate therefore the influence, in data

assimilation, of an observation close to the boundary of the domain on the opposite part of the domain.

4. Conclusions

We have described in this presentations two of the areas of development being carried out in the HIRLAM consortium to improve the HARMONIE limited area forecast model.

In the first one we have described the adaptation to the non-hydrostatic version, of the finite-element procedure in the vertical implemented some time ago for the hydrostatic version. This adaptation is also applicable to the global version (IFS), which will be tested in collaboration with ECMWF.

In the second, we describe the elimination of the extension zone from the grid-point representation in the forecast model, allowing then, with very little increase of the computational cost, to increase the width of this zone in order to avoid the unrealistic influence, in data assimilation, of an observation close to the boundary of the domain, on the opposite boundary of the domain.

5. References

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