Diagnosing Remote Origins of Forecast Error and Circulation Anomalies Using Relaxation Experiments

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ABSTRACT

In this article a method is introduced that can be used to diagnose possible remote origins of forecast error and circulation anomalies. This is achieved by relaxing the model towards (re-)analysis data in certain regions during the course of the integration and, thereby, suppressing the development of forecast error. The method is illustrated for two different applications. Firstly, experiments are carried out to study the influence that a correct representation of the tropical atmosphere and the Northern Hemisphere stratosphere would have on extended-range forecast skill of the extratropical Northern Hemisphere troposphere during boreal winter. Secondly, the origin of the atmospheric circulation anomalies, that led to the unusually cold European winter of 2005/06, is studied. Finally, the relaxation approach is tested compared to a more sophisticated 4D-Var data assimilation approach.

1 Introduction

Despite substantial improvements in model formulation, data assimilation systems and observing systems, forecasts are still prone to failures. This is particularly true for extended-range forecasts (beyond 10 days) of the extratropical flow, which have moderate skill at the best of times. Apart from being of scientific interest, understanding in which regions forecast error originates is a first step towards future forecasting system improvements. If it would turn out, for example, that extended-range preditive skill in the extratropics is primarily limited by model error in the tropics then future model development should focus on exactly this region.

Here it is shown that a relaxation technique (also sometimes called nudging) can be used to diagnose possible 'remote' origins of forecast error. In this diagnostic technique the model's prognostic fields are relaxed towards analysis data in certain parts of the globe during the course of the 'forecast'. In this way it is possible to suppress artificially the development of forecast errors in certain regions of the globe such as the tropics and study possible impacts on predictive skill in remote regions such as the Northern Hemisphere.

The relaxation technique is a well-established technique in the atmospheric sciences. It has been used, for example, in data assimilation (see Kalnay, 2003, for an overview), for determining corrections to empirically reduce model deficiencies (Kaas et al., 1999), for dynamical downscaling (von Storch et al., 2000), for better understanding planetary wave–synoptic wave interactions in the atmosphere (Straus and Yi, 1998), and for validation of a synoptic system in an atmospheric circulation model (Bauer et al., 2008). The approach employed in this study is very similar to the method used at ECMWF in the 1980s in order to understand the origin of *medium-range* forecast error in the northern hemisphere extratropics (Haseler, 1982; Klinker, 1990; Ferranti et al., 1990). It has been decided to revive the relaxation technique at ECMWF as a diagnostic tool for the following reasons:

- The relaxation technique could also be used to understand forecast error in the extended-range, addressing the monthly and seasonal forecasting problem.

- The availability of larger computer resources allows significant increases in sample size and therefore robustness of the results compared to previous studies.

- The availability of more realistic analysis data, particularly in the tropics, makes the relaxation technique much more effective.

In the following it will also be shown that the relaxation technique can help us to understand possible 'remote' origins of extratropical atmospheric circulation anomalies. It is well-known that persistent large-scale extratropical circulation anomalies such as the North Atlantic Oscillation (NAO) have a profound impact on the climate of populated areas such as Europe and North America (e.g. van Loon and Rogers, 1978; Hurrell, 1995). Attempts have therefore been made to understand the mechanisms that drive extratropical atmospheric circulation anomalies. It is now widely accepted that a large part of the extratropical variability in the North Atlantic region is governed by internal atmospheric processes (e.g. Kushnir et al., 2002; Rowell, 1996), especially on seasonal and interannual time scales. This suggests that predictability of such anomalies is limited to a few weeks. There is observational and modelling evidence, however, that the atmosphere in the North Atlantic region is also affected (i) locally by North Atlantic sea surface temperature (SST) anomalies (e.g Czaja and Frankignoul, 1999; Rodwell and Folland, 2002; Rodwell et al., 1999; Latif et al., 2000) and (ii) remotely by tropical SST anomalies via atmospheric teleconnections (e.g. Fraedrich, 1994; Greatbatch and Jung, 2007). Furthermore, it has been suggested that the Northern Hemisphere stratosphere may provide some additional memory which could result in some useful monthly and seasonal forecast skill (e.g. Baldwin et al., 2003; Scaife and Knight, 2008). However, the relative impact of the North Atlantic, the tropics and the extratropical stratosphere has yet to be assessed.

The paper is organized as follows. In section 2, the relaxation technique will be described in some detail. In section 3, the relaxation technique is applied to study possible remote origins of medium-range and especially extended-range forecast error. In section 4, results from the relaxation method are compared with results obtain by using 4D-Var data assimilation with assimilation of data confined to certain regions. This is followed by an investigation of the physical mechanisms giving rise to the anomalously cold European winter 2005/06.

2 Relaxation formulation

The basic idea behind the relaxation technique is to 'artificially' suppress the development of forecast error in certain regions by a adding an extra term of the following form to the ECMWF:

$$-\lambda(\mathbf{x}-\mathbf{x}_{ref}). \quad (1)$$

The model state vector is represented by $\mathbf{x}$ and the reference field towards which the model is drawn by $\mathbf{x}_{ref}$. In this study, $\mathbf{x}_{ref}$ represent analysis data, that is, our best estimate of the true state of the atmosphere. The strength of the relaxation is determined by $\lambda = a \cdot \lambda_0$, where a is a function of longitude, latitude, height and the parameter being considered and λ_0 is a constant. The units of λ are in (time step)$^{-1}$. Unless stated otherwise $\lambda_0 = 0.1\,hrs^{-1}$ is used throughout the study. For a time step of one hour used here a value of $0.1\,hrs^{-1}$ indicates that at each time step the model tendency is 'corrected' using 10% of the departure of $\mathbf{x}$ from $\mathbf{x}_{ref}$. In this study the parameters being relaxed include u, v, T and $\ln p_s$. Notice, that $\ln p_s$ is *not* relaxed for stratospheric relaxation experiments. The 6-hourly reference fields ($\mathbf{x}_{ref}$) were obtained from different analysis and reanalysis data, respectively. They were linearly interpolated in time to 1-hourly values (time step of the model). Spatial interpolation, if necessary, was accomplished by using a sophisticated horizontal interpolation package used routinely within the ECMWF Integrated Forecasting System.

In order to allow for an effective localization, the relaxation was carried out in grid point space. When applying masks to localize the relaxation, care has to be taken in order to reduce adverse effects close to the relaxation boundaries. Here the transition from relaxed to non-relaxed regions in the horizontal is smoothed using the hyperbolic tangent. The smoothing is such that the relaxation coefficient λ goes from λ_0 to zero within a 20^o belt, both in longitude and latitude (see Jung et al., 2010b, their Fig. 1). Boundaries stated in the text refer to the centre of the respective 20^o belt. In order to reduce the generation of spurious potential vorticity features, changes of λ are also smoothed in the vertical (see Jung et al., 2010b, their Fig. 2).

3 Medium-range and extended-range forecast error

Despite substantial improvements in model formulation, data assimilation systems and observing systems, forecasts are still prone to failures. This is particularly true for extended-range forecasts (beyond 10 days) of the extratropical flow, which have moderate skill at the best of times. Apart from being of scientific interest, understanding the origin of forecast error is a first step towards future forecasting system improvements. One important piece of information is the origin of forecast error. If extended-range predictability in the extratropics is primarily limited by model error in the tropics then future model development should focus on exactly this region.

In this section the relaxation technique is employed to estimate how much of the extratropical forecast errorin extended-range (11–30 days) integrations originates from parts of the climate system with (potentially) enhanced extended-range predictability (e.g. Baldwin et al., 2003; Shukla, 1998): the lower bounday conditions, the tropical atmosphere and the stratosphere.

3.1 Experimental setup

To investigate the origin of extratropical forecast error during boreal winter a large set of 30-day control and relaxation experiments has been carried out using model cycle 32r1 (used operationally at ECMWF from 5 June–5 November 2007) at a resolution of T_L159 (about 125 km) and with 60 vertical levels ($T_L159L60$). For each of the experiments a total of 88 30-day forecasts were carried out. Forecasts were started on the 15^{th} of the months November, December, January and February, for each of the winters from 1980/81 to 2001/02. Initial conditions were taken from ERA-40 reanalysis data. If not stated otherwise, sea surface temperature (SST) and sea ice fields were persisted throughout the forecast. An additional control integration with observed SST and sea ice fields from ERA-40 was also carried out in order to quantify the influence that ‘knowledge’ of the lower boundary conditions has on atmospheric forecast skill.

Forecast experiments with relaxation of the following regions towards ERA-40 reanalysis data have been carried out:

- whole tropical atmosphere (TROP),
- tropical stratosphere (TROP-S),
- tropical troposphere (TROP-T),
- Northern Hemisphere stratosphere (NH-S), and
- Northern Hemisphere troposphere (NH-T).

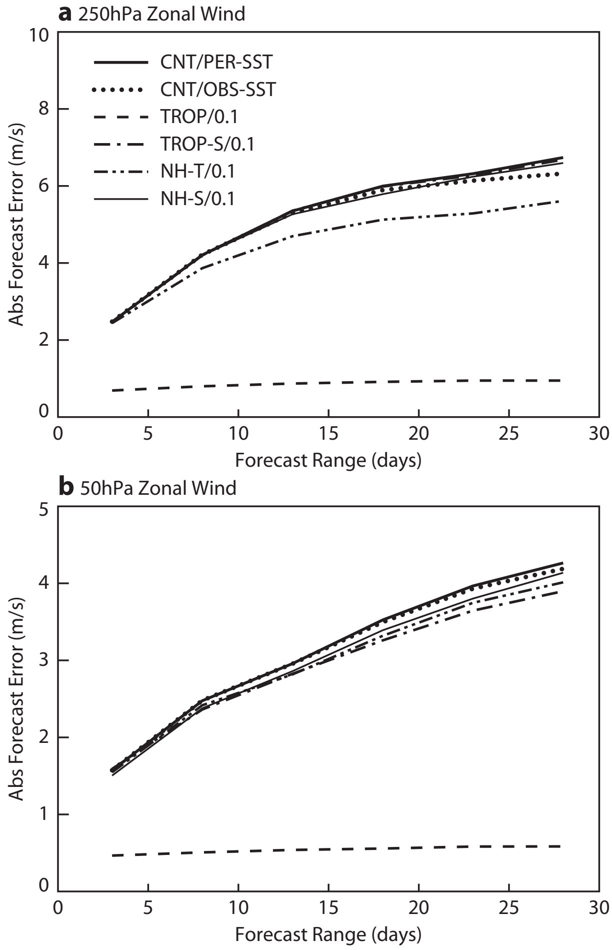

Figure 1: Mean absolute error ($m s^{-1}$) of 5-day averaged forecasts of zonal wind at (a) 250 hPa and (b) 50 hPa. Results are shown for the control forecast with persisted (solid, stream function at 200 hPa in the tropics (10°S–10°N) for control forecast with persisted (CNT/PER-SST) and observed (CNT/OBS-SST) SSTs as well as for experiments with relaxtion towards ERA-40 reanalysis data in the tropics (TROP/0.1), the tropical stratosphere (TROP-S/0.1), the Northern Hemisphere troposphere (NH-T/0.1) and the Northern Hemisphere stratosphere (NH-S/0.1).

Additional sensitivity experiments were carried out to investigate the relative importance of different tropical regions and to study the sensitivity to the strength of the relaxation. Further details are given in Jung et al. (2010b).

3.2 Tropical forecast error

Figure 1 shows mean absolute forecast error of 5-day averaged zonal wind at the 250 hPa (tropical troposphere) and 50 hPa level (tropical stratosphere). The control integration (CNT/PER-SST) shows increasing forecast error in the tropical troposphere throughout the 30-day forecast period suggesting that current forecasting systems possess some useful monthly forecast skill (see also Vitart, 2004). In the tropical stratosphere there is no evidence for saturation of forecast error throughout the first 30 days suggesting a relatively high level of extended-range predictive skill.

Prescribing rather than persisting SST fields throughout the integration (CNT/OBS-SST) reduces forecast error of the tropical troposhere slightly in the extended-range; in the medium-range better 'knowledge' of SST has no impact on forecast skill (Fig. 1a). Not too suprisingly, the influence of the lower boundary conditions has a rather small effect on tropical stratosphere.

The experiment with relaxation of the whole tropical atmosphere (TROP/0.1) shows that the relaxation is efficient in reducing forecast error in both the troposphere and the stratosphere. Throughout the 30-day forecasts, forecast error of zonal wind at 250 and 50 hPa are kept significantly below the level seen in the short-range and early medium-range (5-day average from D+1 to D+5).

The influence of the Northern Hemisphere (NH-S/0.1) and especially the tropical stratosphere (TROP-S/0.1) on tropical zonal winds at 250 hPa is relatively small (Fig. 1a). The largest 'non-local' influence comes from the Northern Hemisphere extratopics, whose impact is felt throughout the whole forecast. This finding is consistent with the notion that extratropical forcing can influence tropical convection and equatorial waves (Kiladis and Weickmann, 1992; Hoskins and Yang, 2000).

Tropical zonal winds at the 50 hPa level (Fig. 1b) are clearly influenced by a better representation of the tropical troposphere. This is expected given that gravity waves and equatorial planetary-scale (Kelvin and Rossby) waves tend to propagate from the trosposhere into the stratosphere (e.g., Baldwin et al., 2001; Ern et al., 2007). The tropical stratosphere is not only influenced from below; both the extratropical troposphere and stratosphere have some impact on the tropical stratosphere.

3.3 Extratropical forecast error

Figure 2 shows mean absolute forecast error of 5-day averaged extratropical Northern Hemisphere[1] geopotential height fields at the 500 hPa level (Z500, hereafter) for various experiments. The control integrations with persisted and observed SST/sea ice fields (CNT/PER-SST and CNT/OBS-SST) show that it takes about 30 days for forecast error to saturate and that knowledge of the lower boundary conditions increases the skill in the extended-range slightly (Figure 2a); in the short-range and medium-range, on the other hand, using observed rather than persisted lower boundary conditions provides little, if any, benefit (see also Jung and Vitart, 2006).

Relaxing the tropics (TROP/0.1) and the Northern Hemisphere stratosphere (NH-S/0.1) both lead to a noteworthy reduction in Z500 forecast error over the Northern Hemisphere (Figure 2a). In relative terms the forecast error reduction is largest in the extended-range (beyond D+10), where it amounts to about 10–20% of the forecast error of the control integration for TROP/1.0 and NH-S/1.0. The 'delayed' positive impact of the tropical and stratospheric relaxation can be explained by the fact that forecasts are still quite successful in the short-range and medium-range (where the relaxation has little work to do). Furthermore, it can be expected to take a few days for the signal (i.e. forecast error reduction) to 'propagate' from the tropics and the stratosphere, respectively, into the northern hemisphere troposphere (e.g. Hoskins and Ambrizzi, 1993; Baldwin and Dunkerton, 1999; Jung and Barkmeijer, 2006).

The sensitivity of the results to the strength of the relaxation (i.e., the choice of λ in Eqn. 1) for TROP and NH-S can be inferred from Figure 2 b and c, respectively. For the relaxation time scales considered here (1, 10 and 50 hours) the tropical relaxation appears to be less sensitive to the choice of λ. One way to interpret this result is that the reduction of Northern Hemisphere Z500 error is due to relatively persistent and large-scale rather than fast and small-scale tropical features. For NH-S, the forecast error reduction for Z500 appears to be relatively more sensitive to λ. One possible way of explaining the fact that a relatively strong relaxation is required for the Northern Hemisphere stratosphere is that stratospheric motions are strongly governed by the underlying troposphere (see below for more details).

As shown above, relaxation of the tropical atmosphere leads to reduced forecast error over the Northern Hemisphere. How much of this improvement originates in the tropical troposphere and how much in

[1]Here the Northern Hemisphere encompasses only the region north of 40^oN in order to stay well clear of the relaxation zone used in experiment TROP/0.1

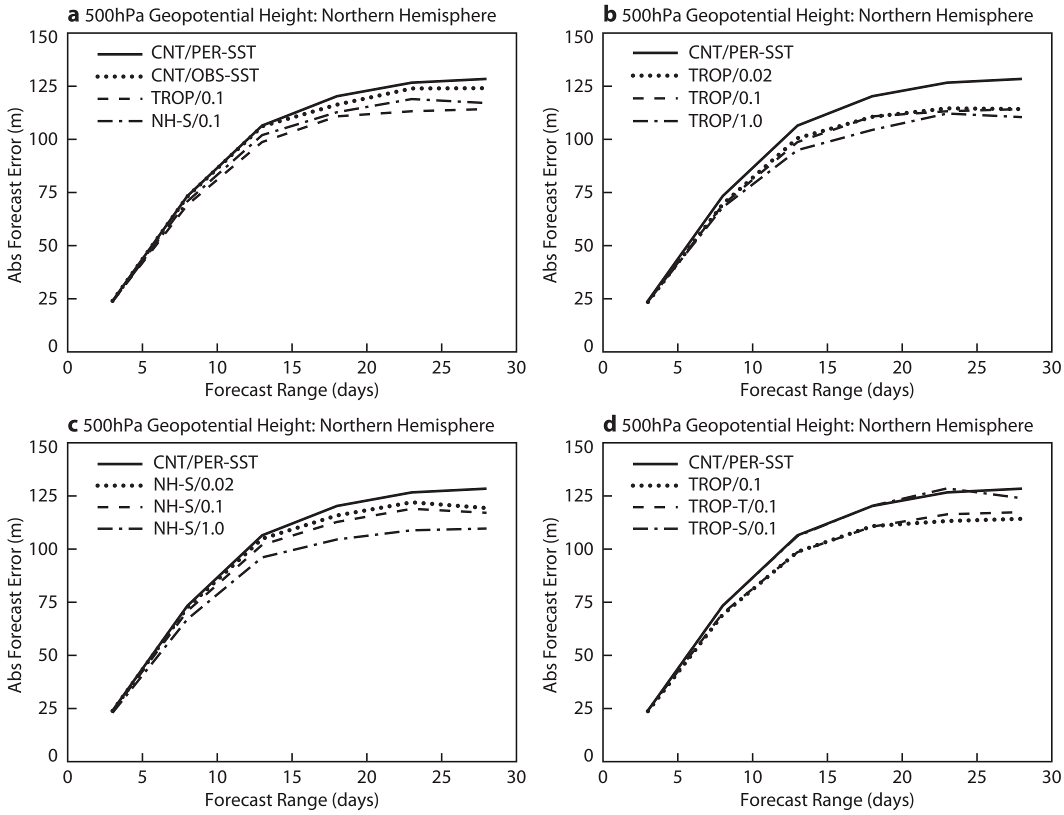

Figure 2: Mean absolute error (m) of 5-day averaged forecasts of 500 hPa geopotential height fields over the Northern Hemisphere (north of 40°N): (a) control forecast with persisted and observed SSTs as well as for experiments with relaxation of the tropics (TROP/0.1) and the Northern Hemisphere stratosphere (NH-S/0.1) towards ERA-40 reanalysis data. (b) as in (a), but for different tropical relaxation experiments (TROP/0.02, TROP/0.1 and TROP/1.0). (c) as in (a), but for different experiments with relaxation of the Northern Hemisphere stratosphere (NH-S/0.02, NH-S/0.1, and NH-S/1.0)

the tropical stratosphere? In order to answer this question, additional relaxation experiments have been carried out with relaxation of the tropical troposphere (TROP-T/0.1) and tropical stratosphere (TROP-S/0.1) only. Results from these experiments clearly show that it is primarily the tropical *troposphere* which influences the tropospheric flow over the Northern Hemisphere (Fig. 2d).

How the relaxation towards ERA-40 in different regions influences the predictability of the *stratospheric* circulation (in terms of geopotential height at 50 hPa, Z50 hereafter) over the Northern Hemisphere can be inferred from Fig. 3. The forecast error of the control integration saturates much later at 50 hPa than it does at 500 hPa. This highlights the relatively high level of extended-range predictability of the Northern Hemisphere stratosphere. The tropics have some influence on the stratospheric circulation, especially beyond D+15 or so. Not too surprisingly, relaxing the stratosphere towards ERA-40 reduces Z50 forecast error over the Northern Hemisphere substantially. Interestingly, however, relaxing the extratropical *troposphere* has a similar influence, at least for values of λ much smaller than 1.0. These results are a reminder of the strong troposheric forcing of the Northern Hemisphere stratosphere during boreal winter.

So far, the focus has been on Z500 forecast error for the extratropical Northern Hemisphere as a whole. It is likely, however, that the Z500 response over the Northern Hemisphere described above shows some interesting spatial structure. Regional influences of how prescribing rather than persisting the lower

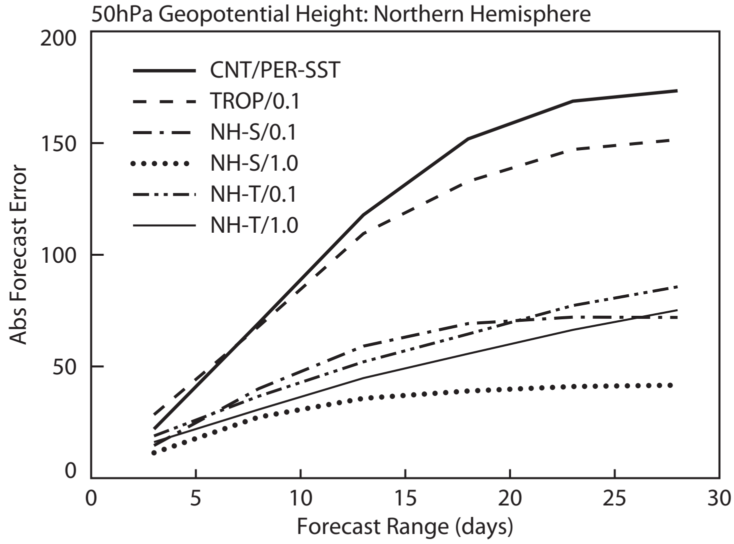

Figure 3: Mean absolute error (m) of 5-day averaged forecasts of 50 hPa geopotential height fields over the Northern Hemisphere (north of 30^{o}N) for control forecast with persisted SSTs (CNT/PER-SST) and experiments with the tropics (TROP/0.1), the northern hemisphere stratosphere (NH-S/0.1 and NH-S/1.0) and the northern hemisphere troposphere (NH-T/0.1 and NH-T/1.0) relaxed towards ERA-40 reanalysis data.

boundary conditions affects Northern Hemisphere Z500 forecast error can be inferred from Figure 4d–f. Perfect knowledge of the observed SST/sea ice fields has a positive impact primarily in the extended-range over the North Pacific and over North America. The impact over the North Atlantic and Europe, on the other hand, is rather small (and not significant) throughout the first 30 days of the forecast.

Not too surprisingly, the tropical relaxation experiment, TROP/0.1 (Fig. 4g–i), leads to substantial forecast error reduction in the northern hemisphere subtropics, that is, close to the relaxation region. The fact that the forecast error reduction with tropical relaxation appears to be largely ‘confined’ to the subtropics in certain regions such as south-east Asia might be explained by the presence of strong subtropical wave guides (e.g. Branstator, 2002) which convey the energy is zonal rather than meridional direction. There is also a clear positive impact of a correct representation of the tropics in certain regions of the Northern Hemisphere *mid-latitudes* such as the eastern North Pacific, the North American continent and the central North Atlantic. This is true from the medium-range well into the extended-range. In the Euro-Atlantic region the Z500 forecast error reduction is largest in the eastern North Atlantic. This is an area which is known for the frequent occurrence of persistent ridges (‘blocking’) and troughs, both which tend to produce high-impact weather over Western Europe (e.g. UK floods in autumn 2000). North America is the other populated area in the Northern Hemisphere mid-latitudes which benefits from improved forecasts of the tropical troposphere.

In the medium-range and extended-range, the stratospheric relaxation experiment leads to the largest forecast error reduction in high latitudes (Fig. 4j–l). This is consistent with the tropospheric response found in the ECMWF model as a result of changes in the strength of the stratospheric polar vortex (Jung and Barkmeijer, 2006). Interestingly, Europe and northern parts of North America are also key-beneficiaries of a better representation of the stratospheric circulation, both in the medium-range and extended-range.

It is worth mentioning that the *spatial structure* of the response is much less sensitive to the exact choice of λ than is the *magnitude* (not shown).

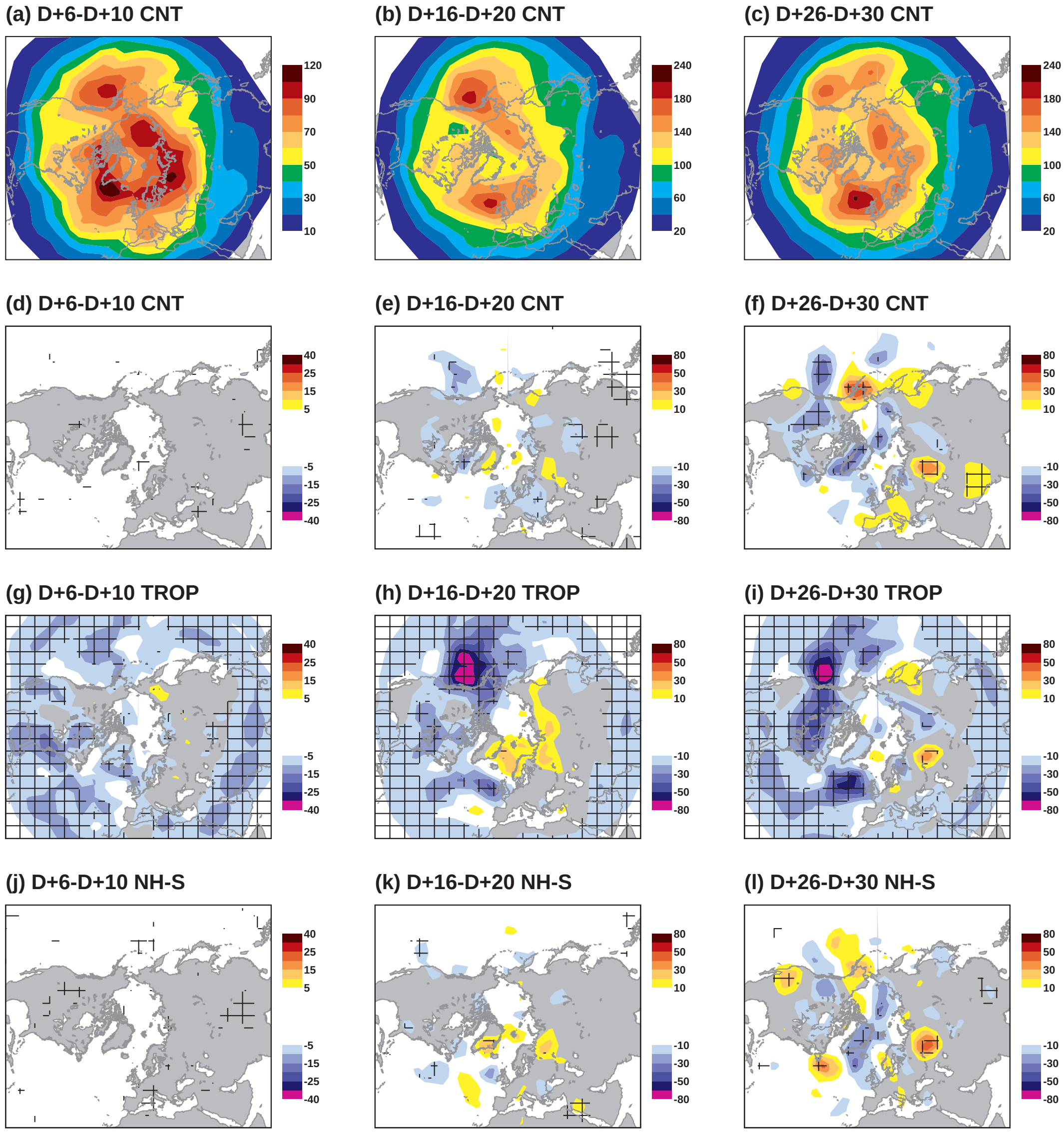

Figure 4: (a)–(c) Mean absolute forecast error of 500 hPa geopotential height field (in metres) for the control integration with persisted SSTs (CNT/PER-SST). (d)–(f) Difference in mean absolute forecast error for Z500 between the control intregration with observed (CNT/OBS-SST) and persisted (CNT/PER-SST) SSTs. (g)–(i) as for (d)–(f), but for the differenced between TROP/0.1 and CNT/PER-SST. (j)–(l) as for (d)–(f), but for the differenced between NH-S/0.1 and CNT/PER-SST. Results are shown for 5-day averaged data: D+6 to D+10 (left), D+16 to D+20 (middle) and D+26 to D+30 (right). Differences significant at the 95% confidence level (two-sided t-test) are hatched.

The same experiments deascribed above were repeated for the *independent* period 1958–1981 (not shown). In general the conclusions remain unchanged, except for a small reduction of the tropical and stratospheric impact on Z500 forecast error over North America. This may at least partly be explained by the slightly poorer quality of the ERA-40 reanalysis during the pre-satellite era (Uppala et al., 2005).

3.4 Summary and discussion

The origin of extended-range forecast error has been studied with the ECMWF model by carrying out relaxation experiments. By spatially confining the relaxation it is possible to study the *remote* impact of forecast error reduction in certain regions. The focus of this study has been on the influence that the tropics and the Northern Hemisphere stratosphere have on extended-range forecast skill of the Northern Hemisphere circulation. Emphasis has been put on the role of the tropics since it is widely believed that extended-range predictions of the extratopical atmosphere benefit from better forecasts of the MJO (e.g. Ferranti et al., 1990; Jones et al., 2004; Moncrieff et al., 2007); the influence of the Northern Hemisphere stratosphere has been studied in more detail in order to understand the role that anomalies in the strength of the stratospheric polar vortex and their 'downward propagation' into the the troposphere (Baldwin and Dunkerton, 2001; Baldwin et al., 2003) have on extended-range forecast skill.

Our results show that a reduction of forecast error in the tropical troposphere has a beneficial impact on extended-range forecast skill over the Northern Hemisphere. In terms of populated areas this is especially true for North America and Western Europe.

The relaxation experiments presented in this subsection were carried out in order to guide future forecasting system development. The tropical relaxation experiments, for example, provide some idea how much forecast skill, if any, could be gained by reducing forecast error in tropics (e.g., by a better representation of physical processes). Our results suggest that reduced tropical forecast error is unlikely to increase extended-range skill in predicting the Northern Hemisphere tropospheric circulation beyond the current skill in the range from D+11–D+15 (Fig. 2a). Notice, however, that there a large regional variations. These estimates have to be seen as rather *optimistic* given that in these experiments tropical forecast error is reduced to levels unlikely to be achieved in the future.

Stratospheric relaxation experiments show that reduced forecast error in the Northern Hemisphere stratosphere leads to reduced forecast error in the troposphere below. These results are consistent with previous modeling studies in which a relatively strong tropospheric response has been found to imposed stratospheric perturbations (e.g. Boville, 1984; Charlton et al., 2004; Jung and Barkmeijer, 2006). However, the stratospheric relaxation experiments are very difficult to interpret in terms of the implied gain in tropospheric predictability. This is because tropospheric relaxation is as efficient in reducing stratospheric forecast error as is direct stratospheric relaxation, highlighting the strong influence of the troposphere on the Northern Hemisphere stratospheric during boreal winter (see also, e.g., Martius et al., 2009). A very illuminating discussion of difficulties in interpreting numerical experiments, in which a strongly forced component of the coupled system is artificially prescribed, is given by Bretherton and Battisti (2000) for the atmosphere-ocean system[2].

Our conclusions are very similar to that from the study by Newman and Sardeshmukh (2008) using a completely different approach by diagnosing linear inverse models fitted to observational data. They find that tropical influences are generally larger than stratospheric influences in terms of predictability of the extratropical troposphere during boreal winter.

4 Testing the relaxation approach using 4D-Var data assimilation

One of the potential weaknesses of the tropical relaxation experiments is the presence of the transition zones in which the strength of the relaxaton coefficient changes. It could be argued, for example, that the presence of the transition zone leads to spurious reflection of extratropical Rossby waves. Furthermore, imbalances may occur within and close to the transition zones.

It order to reduce possible problems associated with imbalanced, here we test the relaxation approcah by comparing it with results from experiments with the ECMWF 4D-Var data assimilation system in which observervation are assimilated in the tropics only.

[2]The atmosphere and ocean in their study correspond to the troposphere and stratosphere, respectively, discussed here.

4.1 Experimental setup

For the purpose of testing the relaxation approch the ECMWF Integrated Forecast System (IFS) is used at a horizontal resolution of T_L159 with 91 levels in the vertical. All experimentation is based on version 33R1, which was used operationally at ECMWF from 3 June to 29 September 2008.

Forecasts experiments were carried out every third day during the period 3 January to 7 March 2009 (a total of 22 forecasts). This period has been chosen for three reasons. Firstly, it samples the winter season when tropical-extratropical interaction are at its strongest. Secondly, it covers the winter component of the THORPEX Pacific Asian Regional Campaign (Winter T-PARC). Winter T-PARC is a multi-national field campaign that addresses the shorter-range dynamics and forecast skill of the Eastern Asian and the western North Pacific region and its downstream impact on the medium-range dynamics and forecast skill downstream over the eastern North Pacific and North America. Thirdly, an area of enhanced convective activity associated with the MJO propagated from the Indian ocean into the tropical Pacific from late January to the beginning of February 2009. Given that the MJO is generally believed to have a noticable impact on the Northern Hemisphere extratropical circulation (Ferranti et al., 1990; Jones et al., 2004) it can be expected that tropical-extratropical interaction does play a role during the period chosen.

In the relaxation experiments the model is drawn towards interpolated operational analysis data in the tropics (20^oS–20^oN) using a value of $\lambda = 0.1\,hrs^{-1}$ in Eqn. 1. The relaxation experiments were augmented by a set of control forecasts without relaxation, which, like the relaxation experiments, were initialized from a low-resolution version of the operational ECMWF analysis.

In this study the ECMWF 4D-Var data assimilation is used (Rabier et al. 2000). The non-linear forecasts are carried out at T_L159 with 91 levels in the vertical; the first and second inner loop employ are resultion of T42 and T_L95, respectively. Variational bias correction is used (Dee 2005, Auligné et al. 2007). The length of the data assimilation window is 6-hours.

In order to test possible remote influences of reduced forecast error in certain areas 'forecasts' were produced by running 4D-Var data assimilation cycles with assimilation of data restricted to certain regions. Like for the relaxation experiment, here the assimilation of data in the tropics only is considered (TROP, 20^oS–20^oN). Data assimilation cycles were run or each of the 22 starting dates and assimilation regions. As control forecast the first guess started from the initial conditions based on the low-resolution T_L159 analysis and extended to 360 hours was used.

4.2 Results

Mean squared anomaly correlation coefficients (ACCs) for 500 hPa geoptential height forecasts in the Northern Hemisphere mid-latitudes are shown in Figure 5a,b for the tropical 4D-Var and relaxation experiments. Most importantly, the diagnosed role of forecast error in the tropics on forecast skill in the Northern Hemisphere mid-latitudes is very similar for both methods. Generally, reduced tropical forecast error leads to better mid-latitude forecasts only beyond D+5 or so. For both types of experiments, however, the 'forecast improvements' beyond D+5 is moderate amounting to about 12–24 hours gain in forecast skill.

One might ask whether the fact that the forecast error reduction in the Northern Hemisphere mid-latitudes associated with a better representation of the tropics is moderate at best is due to the fact the extratropical changes introduced by modifying the tropics are small compared to forecast error (i.e. small perturbation size) or whether the perturbations just do not grow into the right direction to reduce extratropical forecast error. To address this question Figure 5c,d show mean absolute forecast error and the mean absolute forecast differences between 'forced' (tropical observations and relaxation, respectively) and control experiments for Z500 over the Northern Hemisphere mid-latitudes. Henceforth, the former and latter shall simply be referred to as 'forecast error' and 'evolved perturbation', respectively.

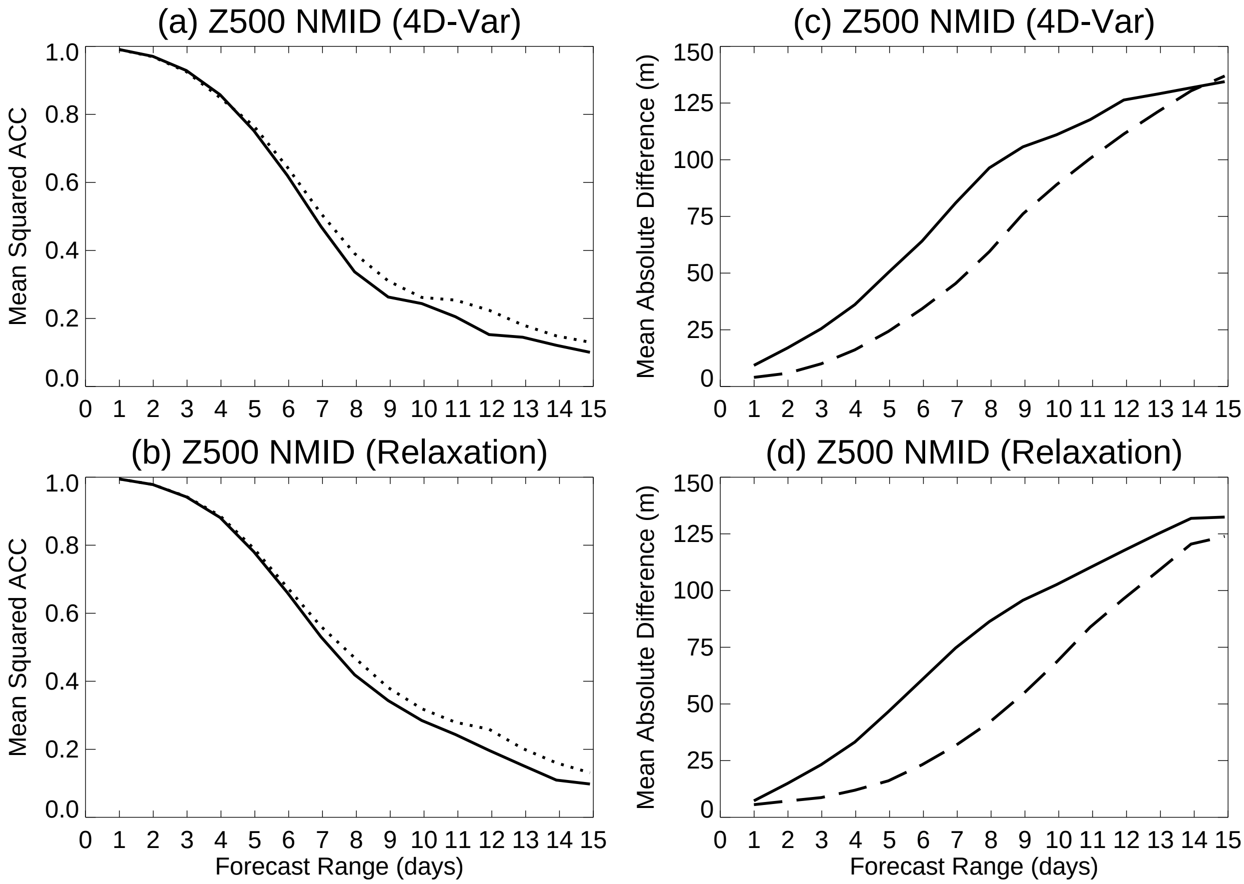

Figure 5: Mean of squared anomaly correlation coefficients between 500 hPa geopotential height forecasts and verifying analyses in the Northern Hemisphere mid-latitudes (40^o–60^{o}N): (a) 4D-Var assimilation experiment with tropical observations only (20^{o}S–20^{o}N, dotted line) with corresponding control forecast (solid line) and (b) tropical relaxation (dotted line) with corresponding control experiment (solid line). Also shown are mean absolute forecasts errors of the control integration (solid) and mean absolute forecast differences (experiment minus control, dashed) for Z500 fields in the Northern Hemisphere mid-latitudes: (c) 4D-Var data assimilation and (d) relaxation experiment. Results are based on all forecasts started every third day during the period 3 January to 7 March 2009 (i.e., a total of 22 forecasts).

Figure 5c,d reveal that for both experiment types the size of the evolved perturbations is much smaller than that of the forecast errors, at least in the short-range and medium-range. From D+10 or so the size of the evolved perturbations becomes comparable to the size of the forecast error. Given that in this forecast range assimilation of data in the tropics as well as tropical relaxation leads to rather small forecast error reductions (Figure 5a,b) suggests that forecast error is primarily governed by extratropical processes.

The control integrations for the two types of experiments also provide some interesting insights. Firstly, the control forecast for the relaxation experiment, on average, is more skillful than the corresponding forcasts from the 4D-Var experiment (Figure 5). This suggests that horizontal resolution during the data assimilation processes is crucial, presumably due to a more realistic fist guess, and, hence, better data usage. Secondly, there is certainly evidence for predictive skill (10–20% explained variance) of Z500 fields in the Northern Hemisphere mid-latitudes up to day, even for a relatively low-resolution model employed in this study.

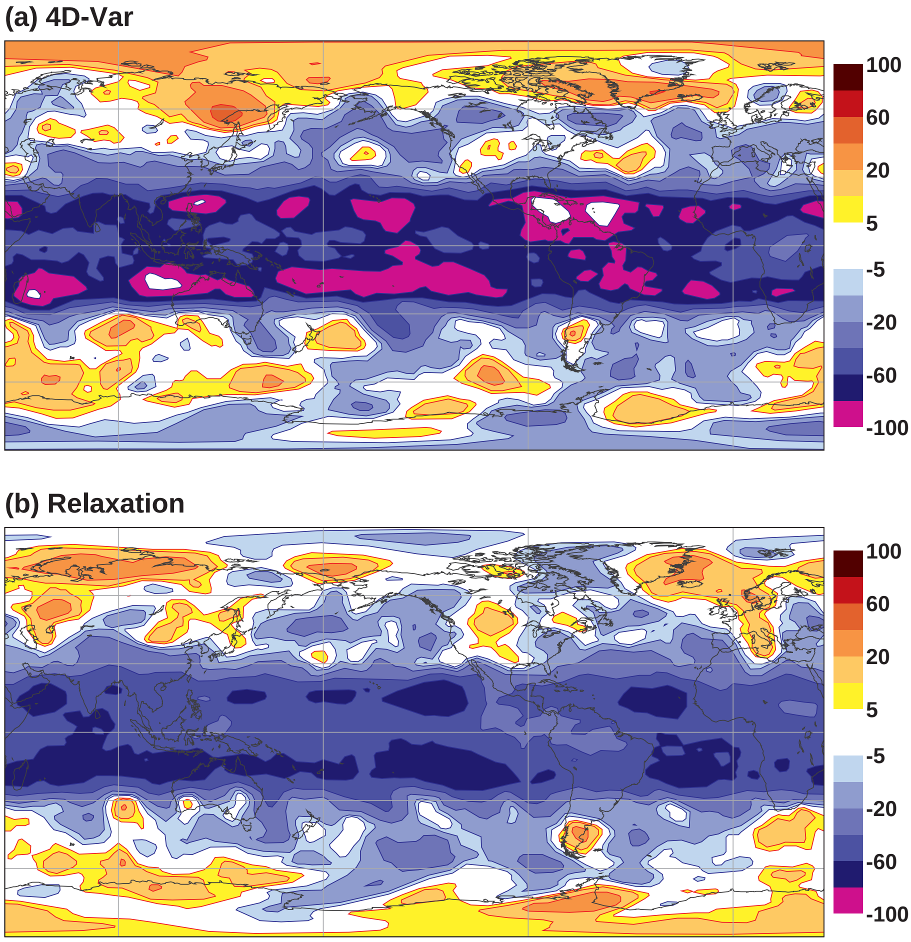

Figure 6: Relative reduction in mean absolute error (shading in %) of D+10 to D+15 forecasts of 500 hPa geopotential height compared to the respective control forecast: (a) 4D-Var assimilation experiment with assimilation of observations in the tropics only and (b) the tropical relaxation experiment. Results are based on forecasts started every third day during the period 3 January to 7 March 2009 (i.e., a total of 22 forecasts).

The way how the assimilation of tropical data as well as tropical relaxation reduces Z500 forecast in different parts of the globe can be inferred from Figure 6 for D+10 to D+15 forecasts. Substantial relative forecast error reduction in the tropics shows the efficiency of the two methods employed. Generally, the picture in the extratropics in somewhat noisy, which reflects the fact that the results are based on only 22 forecasts. However, there are some areas which appears to benefit especially from reduced tropical forecast error for both methods (4D-Var and relaxation), especially for D+10 to D-15 forecasts, namely the North Pacific, North America, the North Atlantic and Europe on the Northern Hemisphere as well as the South Pacific and South Atlantic on the Southern Hemisphere.

4.3 Summary and discussion

Tropical relaxation experiments have been compared with results from 4D-Var data assimilation experiments with tropical observations only. This comparison has been carried out to understand whether potential weaknesses of the relaxation approach such as imbalances close to relaxation boundaries and arbitrary choices for relaxation coefficients do have an influence on the conclusions drawn with the relaxation approach. Tropical relaxation has been found to lead to very similar results to 4D-Var experiments with assimilation of tropical observations only. Similar results have been obtained for a region in the extratropics (not shown here). This suggest that the relaxation approach is a relatively cheap (two orders of magnitude less expensive than 4D-Var) and efficient diagnostic technique to study remote orgins of forcast error.

5 The anomalously cold European winter of 2005/06

The relaxation technique has been widely used by the atmospheric science community on relatively shorter 'weather' time scales (Kalnay, 2003; Bauer et al., 2008). Here it will be illustrated as a diagnostic tool to understand processes on longer seasonal and climatic time scales.

To this end the anomalously cold European winter of 2005/06 makes an interesting case study for various reasons. Firstly, it was the coldest winter in Europe in about a decade (Scaife and Knight, 2008), which was brought about by an increased frequency of occurrence of Euro-Atlantic blocking events (Croci-Maspoli and Davies, 2009). This increase becomes apparent in the form of a seasonal-mean anti-cyclonic anomaly in geopotential height fields at the 500hPa level (hereafter Z500) over the North Atlantic. Secondly, most seasonal forecasting system showed some skill in predicting the anomalously cold temperatures several months in advance (Graham et al., 2006; Folland et al., 2006) suggesting that some external forcing (or *slowly* varying internal dynamics) might have played a role. Finally, the winter of 2005/06 was marked by the presence of a number of climate anomalies, both in the Northern Hemisphere extratropics and in the tropics, which might explain the observed circulation anomaly (Jung et al., 2010c).

5.1 Experimental setup

The numerical experimentation desribed in this section is based on a recent version of the ECMWF atmosphere model (cycle 32R1 used operationally from 5 June to 5 November 2007). All forecast experiments employ a horizontal resolution of T_L95 (linear Gaussian grid $\approx 1.85^o \times 1.85^o$) with 60 levels in the vertical. About half of the levels are located above the tropopause (Untch and Simmons, 1999) extending up to 0.1 hPa. All experiments were carried out using initial conditionsm, lower boundary conditions (SST and sea ice) and reference fields towards which the model is relaxed from ERA-Interim. Aspects of the model's performance are discussed elsewhere (Jung, 2005; Jung et al., 2010a).

The various relaxation experiments described in this section and their abbreviations are summarized in Table 1.

For the winter of 2005/06 a set of seasonal ensemble forecasts with and without relaxation was carried out using a lagged approach. The ensembles were generated by starting forecasts in 6-hourly intervals from 12 UTC on 16 November to 12 UTC on 20 November 2005 giving a total of 17 ensemble members. A summary of all seasonal forecast experiments along with their abbreviations is given in Table 1.

Throughout this paper 'anomalies' refer to departures of the ensemble mean or individual ensemble members from the climate of the model. The climate of the model was obtained from calibration runs. These runs are single integrations (i.e. one ensemble member) covering winters of the period 1990/91 to 2005/06. Forecasts were started at 12UTC on 15 November. Notice, that the calibration runs for the relaxation experiments were carried out with the same relaxation as for the respective winter 2005/06 ensemble experiment. By doing this, the 'anomalies' reflect the anomalous conditions during the 2005/05 winter rather than the remote influence of reduced systematic errors in the relaxation regions.

Table 1: Summary of the main seasonal forecast experiments used in this section. Unless mentioned otherwise, $\lambda = 0.1 hrs^{-1}$ is used throughout.

Experiment	Relaxation Region	
CNT	no relaxation	—
TROP	20^oS–20^oN, 0^o–360^oE	troposphere+stratosphere
TROP-T	20^oS–20^oN, 0^o–360^oE	troposphere*
TROP-S	20^oS–20^oN, 0^o–360^oE	stratosphere†
TROP-T/30–90E	20^oS–20^oN, 0^o–90^oE	troposphere*
TROP-T/150E–120W	20^oS–20^oN, 150^oE–120^oW	troposphere*
TROP-T/90W–0	20^oS–20^oN, 90^oW–0^o	troposphere*
NH	30^oN–90^oN, 0^o–360^oE	troposphere+stratosphere
NH-S	20^oN–90^oN, 0^o–360^oE	stratosphere†

* Actual strength of the relaxation at 500, 200, 50 and 20 hPa is approximately $\lambda_0 \cdot 0.999, \lambda_0 \cdot 1.8 \cdot 10^{-2}, \lambda_0 \cdot 8.3 \cdot 10^{-7}$ and $\lambda_0 \cdot 1.5 \cdot 10^{-8}$ hrs^{-1}, respectively.
† Actual strength of the relaxation at 500, 200, 50 and 20 hPa is approximately $\lambda_0 \cdot 1.1 \cdot 10^{-7}, \lambda_0 \cdot 2.3 \cdot 10^{-6}, \lambda_0 \cdot 1.8 \cdot 10^{-2}$ and $\lambda_0 \cdot 0.5$ hrs^{-1}, respectively.

5.2 Results

Observed Z500 anomalies for the 2005/06 winter are shown in Figure 7 alongside corresponding anomalies for the control experiment with observed SST/sea ice (CNT), the tropical relaxation experiment (TROP) and the experiment with relaxation of the Northern Hemisphere stratosphere (NH-S). Figure 7b shows that prescribing the observed SST/sea ice fields is not sufficient to reproduce the observed circulation anomalies in an ensemble mean sense, especially over North America, the North Atlantic and Europe. The Z500 response produced by TROP is highly significant and resembles the negative phase of the Arctic Oscillation/North Atlantic Oscillation (AO/NAO) (Thompson and Wallace, 1998; Walker, 1924). Especially over North America, the North Atlantic and Europe, the ensemble mean response to tropical relaxation closely resembles the observed anomalies. The influence of the Northern Hemisphere stratosphere, NH-S, on Northern Hemisphere Z500 anomalies is weaker and different in terms of its spatial structure compared to that from the tropics. The Northern Hemisphere Z500 response for NH-S shows a significant anti-cyclonic circulation anomaly in the eastern North Atlantic, which shows little resemblance to the AO/NAO-like response expected to arise from the 'downward propagation' of polar vortex anomalies (e.g. Baldwin and Dunkerton, 1999; Ambaum and Hoskins, 2002; Jung and Barkmeijer, 2006). That the ensemble-mean responses are very different for TROP and NH-S can be inferred from the fact that spatial pattern correlation coefficients between the two fields, for both the Northern Hemisphere ($r = 0.1$) and the Euro-Atlantic region ($r = 0.3$), are very small.

So far, the results suggest that primarily the tropical anomalies and secondarily the anomalously weak stratospheric polar vortex contributed to the tropospheric circulation anomalies observed during the 2005/06 winter. Figure 8 shows observed 50 hPa geopotential height (Z50) anomalies; also shown are ensemble mean anomalies for CNT and TROP. The Z50 anomalies produced by NH-S are very similar to the observations (not shown). CNT shows weak and non-significant Z50 anomalies suggesting that the observed SST and sea ice anomalies have contributed little to the anomalously weak stratospheric polar vortex. The ensemble mean for TROP, on the other hand, produces a weakened stratospheric polar vortex, with an anomaly which is stronger than observed. Inspection of the individual ensemble members (not shown) suggests that the stratospheric response to a tropical forcing is consistent with the observations. These results suugest that the origin of anomalously weak stratospheric vortex during the

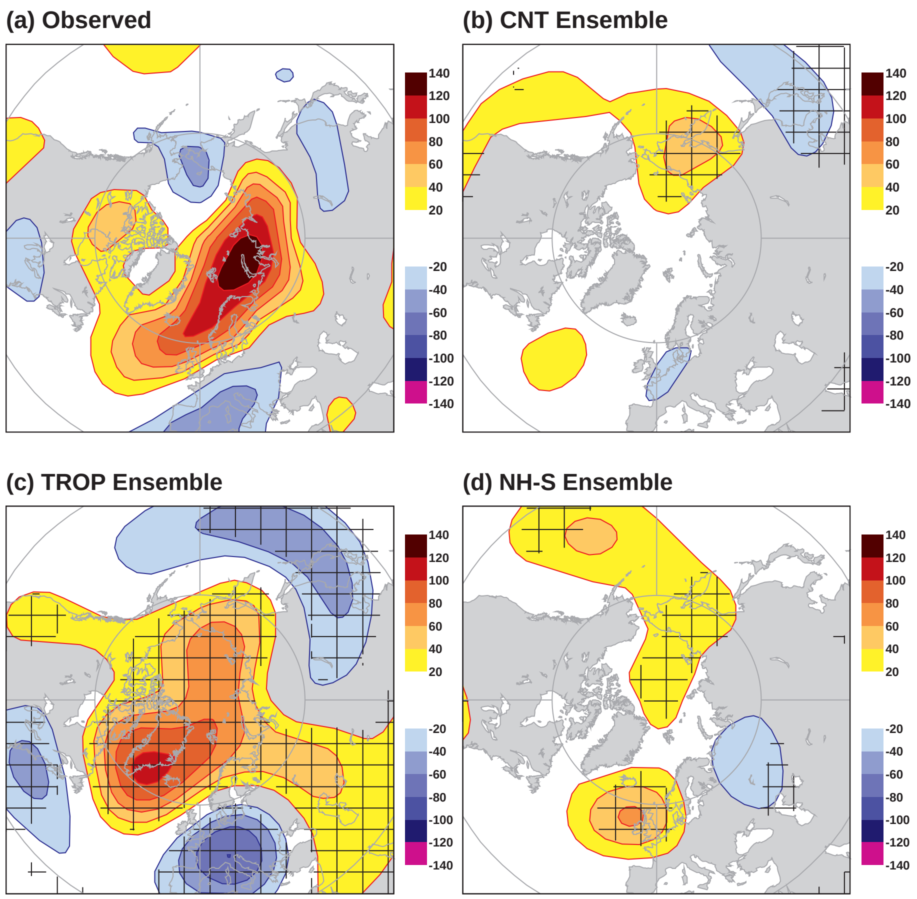

Figure 7: Geopotential height anomalies at the 500 hPa level (contour interval is 20 m) for the period 1 December 2005 to 28 February 2006: (a) ERA Interim, (b) CNT ensemble, (c) TROP ensemble and (d) NH-S ensemble. Results in (b)–(d) are based on ensemble mean data. Statistically significant anomalies (at the 95% confidence level) in (b)–(d) are hatched.

2005/06 winter lies in the tropics

5.3 Summary and discussion

Numerical experiments with the ECMWF model have been carried out in order to understand the origin of the atmospheric circulation anomaly that led to the anomalously cold European winter of 2005/06. In contrast with most other previous studies, which explain observed atmospheric circulation anomalies primarily in terms of SST anomalies in the extratropical North Atlantic (Graham et al., 2006; Folland et al., 2006; Scaife and Knight, 2008; Croci-Maspoli and Davies, 2009), the relaxation experiments presented in this study indicate an important role for the tropical atmosphere. Scaife and Knight (2008) argue that the January 2006 sudden stratospheric warming is likely to have contributed to the colder 2005/06 winter. While it cannot be excluded that the extratropical stratosphere might have increased the persistence of the cold spell, the results of this study suggest that the origin of the sudden stratospheric warming in January lies in the tropics.

Further results of this study, described in Jung et al. (2010c), suggests that the largest forcing came from the tropical troposphere over South America, the Atlantic and Africa. Their results further suggest that the easterly phase of the QBO also contributed to the observed circulation anomalies, especially in the Northern Hemisphere stratosphere.

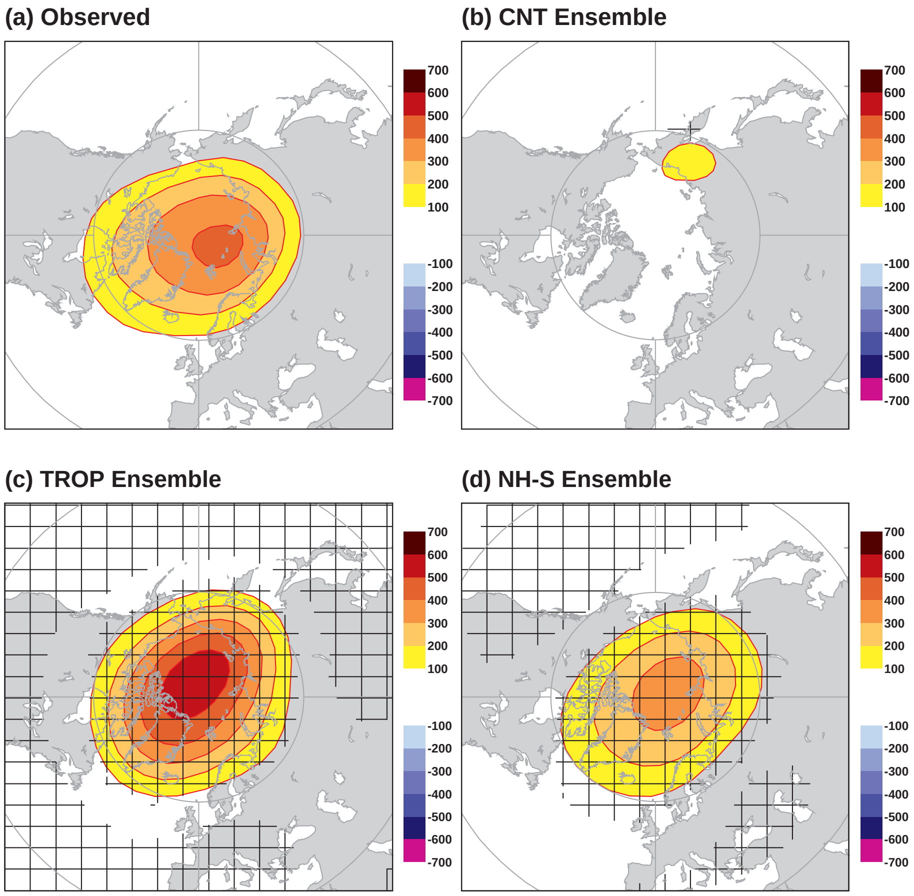

Figure 8: As in Fig. 7, but for 50 hPa Geopotential height anomalies.

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References

Ambaum, M. H. P. and B. J. Hoskins, 2002: The NAO troposphere-stratosphere connection. *J. Climate*, **15**, 1969–1978.

Baldwin, M. P. and T. J. Dunkerton, 1999: Propagation of the Arctic Oscillation from the stratosphere to the troposphere. *J. Geophys. Res.*, **104**, 30937–30946.

Baldwin, M. P. and T. J. Dunkerton, 2001: Stratospheric harbingers of anomalous weather regimes. *Science*, **294**, 581–584.

Baldwin, M. P., L. J. Gray, T. J. Dunkerton, K. Hamilton, P. H. Haynes, W. J. Randel, J. R. Holton, M. J. Alexander, I. Hirota, T. Horinouchi, D. B. A. Jones, J. S. Kinnersley, C. Marquart, K. Sato, and M. Takahashi, 2001: The Quasi-Biennial Oscillation. *Rev. Geophys.*, **39**, 179–229.

Baldwin, M. P., D. B. Stephenson, D. W. J. Thompson, T. J. Dunkerton, A. J. Charlton, and A. O'Neill, 2003: Stratospheric memory and skill of extended-range weather forecasts. *Science*, **301**, 636–640.

Bauer, H.-S., V. Wulfmeyer, and L. Bengtsson, 2008: The representation of synoptic-scale weather system in a thermodynamically adjusted version of the ECHAM4 general ciruclation model. *Meteorol. Atmos. Phys.*, **99**, 129–153.

Boville, B. A., 1984: The influence of the polar night jet in the tropospheric circulation in a GCM. *J. Atmos. Sci.*, **41**, 1132–1142.

Branstator, G., 2002: Circumglobal teleconnections, the jet stream waveguide, and the North Atlantic Oscillation. *J. Climate*, **15**, 1893–1910.

Bretherton, C. S. and D. S. Battisti, 2000: An interpretation of the results from atmospheric general circulation models forced by the time history of the observed sea surface temperature distribution. *Geophys. Res. Lett.*, **27**, 767–770.

Charlton, A. J., A. O. O'Neill, W. A. Lahoz, and A. C. Massacand, 2004: Sensitivity of tropospheric forecasts to stratospheric initial conditions. *Quart. J. Roy. Meteor. Soc.*, **130**, 1771–1792.

Croci-Maspoli, M. and H. C. Davies, 2009: Key dynamical features of the 2005/06 European winter. *Mon. Wea. Rev.*, **137**, 664–.

Czaja, A. and Frankignoul, 1999: Influence of the North Atlantic SST on the atmospheric circulation. *Geophys. Res. Lett.*, **26**, 2969–2972.

Ern, M., P. Preusse, M. Krebsbach, M. G. Mlynczak, and J. M. Russell III, 2007: Equatorial wave analysis from SABER and ECMWF temperatures. *Atmos. Chem. Phys*, **7**, 11685–11723.

Ferranti, L., T. N. Palmer, F. Molteni, and E. Klinker, 1990: Tropical-extratropical interaction associated with the 30–60 day oscillation and its impact on medium and extended range prediction. *J. Atmos. Sci.*, **47**, 2177–2199.

Folland, C. K., D. E. Parker, A. A. Scaife, J. J. Kennedy, A. W. Colman, A. Brookshaw, S. Cusack, and M. R. Huddleston, 2006: The 2005/06 winter in Europe and the United Kingdom: Part II—prediction techniques and their assessment against observations. *Weather*, **61**, 337–346.

Fraedrich, K., 1994: ENSO impact on Europe? *Tellus*, **46A**, 541–552.

Graham, R. J., C. Gordon, M. R. Huddleston, M. Davey, W. Norton, A. Colman, A. A. Scaife, A. Brookshaw, S. Cusack, E. McCallum, W. Elliott, K. Groves, D. Cotgrove, and D. Robinson, 2006: The 2005/06 winter in Europe and the United Kingdom: Part I—How the Met Office forecast was produced and communicated. *Weather*, **61**, 327–336.

Greatbatch, R. J. and T. Jung, 2007: Local versus tropical diabatic heating and the winter North Atlantic Oscillation. *J. Climate*, **20**(10), 2058–2075.

Haseler, J., 1982: An investigation of the impact at middle and high latitudes of tropical forecast errors. Technical Report 31, ECMWF, Shinfield Park, Reading, Berkshire RG2 9AX, UK.

Hoskins, B. J. and T. Ambrizzi, 1993: Rossby wave propagation on a realistic longitudinally varying flow. *J. Atmos. Sci.*, **50**, 1661–1671.

Hoskins, B. J. and G. Y. Yang, 2000: The equatorial response to higher-latitude forcing. *J. Atmos. Sci.*, **57**, 1197–1213.

Hurrell, J. W., 1995: Decadal trends in the North Atlantic Oscillation: Regional temperatures and precipitation. *Science*, **269**, 676–679.

Jones, C., D. Waliser, K. Lau, and W. Stern, 2004: The Madden-Julian Oscillation and its impact on Northern Hemisphere weather predictability. *Mon. Wea. Rev.*, **132**, 1462–1471.

Jung, T., 2005: Systematic errors of the atmospheric circulation in the ECMWF forecasting system. *Quart. J. Roy. Meteor. Soc.*, **131**, 1045–1073.

Jung, T., G. Balsamo, P. Bechtold, A. Beljaars, M. Köhler, M. Miller, J.-J. Morcrette, A. Orr, M. J. Rodwell, and A. M. Tompkins, 2010a: The ECMWF model climate: Recent progress through improved physical parametrizations. *Quart. J. Roy. Meteor. Soc.*, p. submitted.

Jung, T. and J. Barkmeijer, 2006: Sensitivity of the tropospheric circulation to changes in the strength of the stratospheric polar vortex. *Mon. Wea. Rev.*, **134**, 2191–2207.

Jung, T., M. Miller, and T. Palmer, 2010b: Diagnosing the origin of extended-range forecast error. *Mon. Wea. Rev.*, **138**, 2434–2446.

Jung, T., T. N. Palmer, M. J. Rodwell, and S. Serrar, 2010c: Understanding the anomalously cold European winter 2005/06 using relaxation experiments. *Mon. Wea. Rev.*, p. submitted.

Jung, T. and F. Vitart, 2006: Short-range and medium-range weather forecasting in the extratropics during wintertime with and without an interactive ocean. *Mon. Wea. Rev.*, **134**, 1972–1986.

Kaas, E., A. Guldberg, W. May, and M. Déqué, 1999: Using tendency errors to tune the parameterisation of unresolved dynamical scale interactions in atmospheric general circulation models. *Tellus*, **51A**, 612–629.

Kalnay, E., 2003: *Atmospheric Modelling, Data Assimilation and Predictability*. Cambridge University Press.

Kiladis, G. N. and K. M. Weickmann, 1992: Extratropical forcing of tropical Pacific convection during northern winter. *Mon. Wea. Rev.*, **120**, 1924–1939.

Klinker, E., 1990: Investigation of systematic errors by relaxation experiments. *Quart. J. Roy. Meteor. Soc.*, **116**, 573–594.

Kushnir, Y., W. A. Robinson, I. Bladé, N. M. J. Hall, S. Peng, and R. Sutton, 2002: Atmospheric GCM response to extratropical SST anomalies: Synthesis and evaluation. *J. Climate*, **15**, 2233–2256.

Latif, M., K. Arpe, and E. Roeckner, 2000: Oceanic control of decadal North Atlantic sea level pressure variability in winter. *Geophys. Res. Lett.*, **27**, 727–730.

Martius, O., L. M. Polvani, and H. C. Davies, 2009: Blocking precursors to stratospheric warming events. *Geophys. Res. Lett.*, pp. doi:10.1029/2009GL038776,L14806.

Moncrieff, M. W., M. A. Shapiro, J. M. Slingo, and F. Molteni, 2007: Collaborative research at the intersection of weather and climate. *WMO Bulletin*, **56**(3), 204–211.

Newman, M. and P. D. Sardeshmukh, 2008: Tropical and stratospheric influences on extratropical short-term climate variability. *J. Climate*, **21**, 4326–4347.

Rodwell, M., D. P. Rowell, and C. K. Folland, 1999: Oceanic forcing of the wintertime North Atlantic Oscillation and European climate. *Nature*, **398**, 320–323.

Rodwell, M. J. and C. K. Folland, 2002: Atlantic air-sea interaction and seasonal predictability. *Quart. J. Roy. Meteor. Soc.*, **128**, 1413–1443.

Rowell, D. P., 1996: Assessing potential seasonal predictability with an ensemble of multidecadal GCM simulations. *J. Climate*, **11**, 109–120.

Scaife, A. A. and J. R. Knight, 2008: Ensemble simulations of the cold European winter of 2005–2006. *Quart. J. Roy. Meteor. Soc.*, **134**, 1647–1659.

Shukla, J., 1998: Predictability in the midst of chaos: A scientific basis for climate forecasting. *Science*, **282**, 728–731.

Straus, D. M. and Y. Yi, 1998: Interactions of synoptic and planetary waves: Scale-dependent forcing of a GCM. *Mon. Wea. Rev.*, **126**, 876–894.

Thompson, D. W. J. and J. M. Wallace, 1998: The Arctic Oscillation signature in the wintertime geopotential height and temperature fields. *Geophys. Res. Lett.*, **25**, 1297–1300.

Untch, A. and A. J. Simmons, 1999: Increased stratospheric resolution. ECMWF Newsletter 82, ECMWF, Shinfield Park, Reading, Berkshire RG2 9AX, UK.

Uppala, S., P. W. Kallberg, A. J. Simmons, U. Andrae, V. Da Costa Bechtold, M. Fiorino, J. K. Gibson, J. Haseler, A. Hernandez, G. A. Kelly, X. Li, K. Onogi, S. Saarinen, N. Sokka, R. P. Allan, E. Andersson, K. Arpe, M. A. Balmaseda, A. C. M. Beljaars, L. van de Berg, J. Bidlot, N. Bormann, S. Caires, F. Chevallier, A. Dethof, M. Dragosavac, M. Fisher, M. Fuentes, S. Hagemann, E. Holm, B. J. Hoskins, L. Isaksen, P. A. E. M. Janssen, R. Jenne, A. P. McNally, J.-F. Mahfouf, J.-J. Morcrette, N. A. Rayner, R. W. Saunders, P. Simon, A. Sterl, K. E. Trenberth, A. Untch, D. Vasiljevic, P. Viterbo, and J. Woollen, 2005: The ERA-40 re-analysis. *Quart. J. Roy. Meteor. Soc.*, **131**, 2961–3012.

van Loon, H. and J. C. Rogers, 1978: The seesaw in winter temperatures between Greenland and Northern Europe. Part I: General description. *Mon. Wea. Rev.*, **106**, 296–310.

Vitart, F., 2004: Monthly forecasting at ECMWF. *Mon. Wea. Rev.*, **132**, 2761–2779.

von Storch, H., H. Langenbeck, and F. Feser, 2000: A spectral nudging technique for dynamical downscaling purposes. *Mon. Wea. Rev.*, **128**, 3664–3673.

Walker, G. T., 1924: Correlation in seasonal variation of weather, IX. *Mem. Indian. Meteor. Dep.*, **24**(9), 275–332.