Data Assimilation Systems: Focus on EnKF Diagnostics

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1. Introduction

A very productive WWRP/THORPEX Workshop on "4D-VAR and Ensemble Kalman Filter Intercomparisons" took place in Buenos Aires, Argentina, 10-13 November 2008, preceded by a widely attended 2-week Intensive Course on Data Assimilation. The Workshop invited lectures, contributed papers, classes and exercises are posted at http://4dvarenkf.cima.fcen.uba.ar/. There is a special collection of papers presented at the Workshop being reviewed for publication in Monthly Weather Review (Herschel Mitchell, editor). The main conclusion of the Workshop was that 4D-Var and EnKF are currently competitive in skill. It was remarkable that neither method was found to be fatally flawed or even significantly inferior to the other in any area, even from a computational point of view. For EnKF, about 40-100 ensemble members were found to be sufficient for all the weather applications, from storm-scale to global. Another important conclusion, brought forward by the careful experiments carried out at Environment Canada (Buehner et al., 2008), is that hybrid methods combining the advantages of both methods were found to be better than either one alone. There are several possible hybrid approaches (see Barker, 2008, invited lecture). Buehner et al. (2008) tested a hybrid where 4D-Var was run with the background error covariance provided by the operational EnKF. They found that it had about 10 hours of advantage in skill in the Southern Hemisphere compared with either 4D-Var or EnKF.

In this paper we focus on EnKF, a less mature approach than 4D-Var, and discuss a number of new tools for EnKF, in most cases by adapting a method already developed for 4D-Var. We show that it is possible to compute a "no-cost" EnKF smoother for each assimilation window and an outer loop, both similar to the corresponding 4D-Var approaches, that make it possible to deal with nonlinear, non-Gaussian perturbations in long windows. An extension of the outer loop in which the smoother is applied not only to the mean but to the ensemble perturbations as well (dubbed "running in place", RIP) is shown to accelerate the spin-up of the EnKF even in the absence of any prior information. Other new potentially useful tools are a coarse resolution EnKF that does not result in a worse analysis, the adaptive estimation of the background error covariance inflation and of observation errors, a simple estimation of model errors, and the sensitivity of analysis and forecast to observations without the adjoint of the model or the data assimilation system. Several of these tools make use of the analysis ensemble weights (transform matrix of the ETKF, Bishop et al., 2001) that allow expressing the analysis as a weighted average of the ensemble forecasts, so the examples are performed using a type of EnKF, the Local Ensemble Transform Kalman Filter (LETKF) for which those weights are available.

2. Local Ensemble Transform Kalman Filter

In the LETKF (Hunt et al. 2007) the forecasts ensembles are computed globally:

$$\mathbf{x}_{n,k}^b = M_n\left(\mathbf{x}_{n-1,k}^a\right),$$

followed by the construction of forecast and observation perturbation matrices:

$$\mathbf{X}^b = \left[\mathbf{x}_1^b - \overline{\mathbf{x}}^b \,|\, ... \,|\, \mathbf{x}_K^b - \overline{\mathbf{x}}^b\right];$$
$$\mathbf{y}_i^b = H(\mathbf{x}_i^b);\ \mathbf{Y}_n^b = \left[\mathbf{y}_1^b - \overline{\mathbf{y}}^b \,|\, ... \,|\, \mathbf{y}_K^b - \overline{\mathbf{y}}^b\right]$$

Note that $\mathbf{P}^{b,a} = \frac{1}{K-1}\mathbf{X}^{b,a}\left(\mathbf{X}^{b,a}\right)^T$ where K is the number of ensemble members. We follow the notation of Ide et al 1997, with the overbar being the ensemble mean. In the LETKF the analysis is performed in a local grid space; for each grid point the localization is determined by the observations that are used in that grid point analysis. The following computations are performed locally, at each grid point. First the analysis error covariance is computed in ensemble space, indicated by a tilde, taking advantage of the fact that in ensemble space $\tilde{\mathbf{P}}^b = \left(K-1\right)^{-1}\mathbf{I}$:

$$\tilde{\mathbf{P}}^a = \left[\left(K-1\right)\mathbf{I} + \mathbf{Y}^{bT}\mathbf{R}^{-1}\mathbf{Y}^b\right]^{-1};\ \mathbf{W}^a = [(K-1)\tilde{\mathbf{P}}^a]^{1/2}$$

Here $\mathbf{W}^a$, the matrix of perturbation weights, is computed using a symmetric square root. The analysis mean increment in ensemble space is given by

$$\overline{\mathbf{w}}^a = \tilde{\mathbf{P}}^a\mathbf{Y}^{bT}\mathbf{R}^{-1}(\mathbf{y}^o - \overline{\mathbf{y}}^b),$$

and is added to each column of $\mathbf{W}^a$ to get the ensemble analyses increment in ensemble space.

The new ensemble analyses in model space are then the columns of $x_n^a = \mathbf{X}_n^b(\mathbf{W}^a + \overline{\mathbf{w}}^a) + \overline{\mathbf{x}}^b$. Gathering the grid point analysis forms the new global analyses. Note that the LETKF outputs the analysis weights $\overline{\mathbf{w}}^a$ and the analysis matrices of perturbation weights or transforms $\mathbf{W}^a$. These weights multiply the ensemble forecasts to give the analyses.

3. No-cost smoother, outer loop and running in place

As suggested in schematic figure 1, a linear combination of model trajectories is also a model trajectory. Thus, if a linear combination of trajectories at t_n, the end of an assimilation window is close to the truth, indicated by the analysis (arrow point, average of the ensemble analysis, open circles), then it should also be close to the truth throughout the assimilation window, at least to the extent that model errors allow (Brian Hunt, personal comm., 2009). Therefore the weights determined at the end of the assimilation window could be used at the beginning of the window to obtain a smoothed analysis (indicated by a cross in the figure). The smoothed analysis (Kalnay et al., 2007b) is closer to the truth than the previous analysis (arrow at t_{n-1}) since they "know" about the future observations within the assimilation window. The "no-cost" smoother (applying the forecast weights found optimal at t_n to the analyses valid at t_{n-1}, the beginning of the assimilation window) has been tested on the QG model of Rotunno and Bao (1996) by Yang et al., 2009a, and on the WRF model (Shu-Chih Yang, personal comm., 2009) and found to consistently give a more accurate analysis than the corresponding filter. An obvious application of the no-cost smoother would be reanalysis, where the ability of having a smoothed analysis that takes advantage of future observations as well as an ensemble that provides estimations of uncertainty is clear.

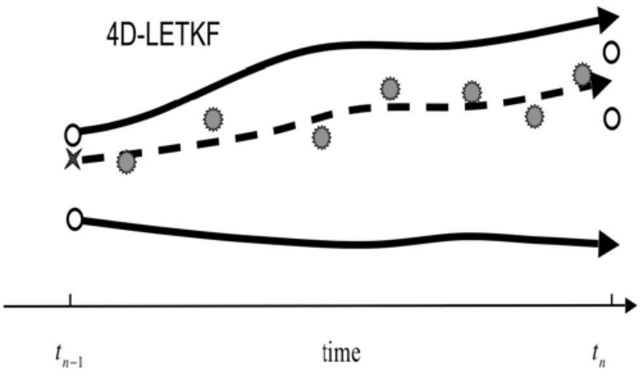

Figure 1: Schematic showing the difference between the LETKF analysis at time t_{n-1}*, indicated by the arrow and given by* $\overline{\mathrm{x}}^a_{n-1} = \overline{\mathrm{x}}^b_{n-1} + \mathrm{X}^b_{n-1}\overline{\mathbf{w}}^a_{n-1}$ *and the LETKF smoother, given by* $\overline{\mathrm{x}}^a_{n-1} = \overline{\mathrm{x}}^b_{n-1} + \mathrm{X}^b_{n-1}\overline{\mathbf{w}}^a_{n}$*, and indicated by a cross. The grey stars are the observations and the open circles the analysis ensemble. Adapted from Kalnay et al., 2007b.*

Computing the smoother at t_{n-1} using the mean weight $\overline{\mathbf{w}}^a_n$ as shown in the schematic figure and retaining the original background error perturbations $\mathbf{W}^a_{n-1}$ re-centers the ensemble about a more accurate solution, allowing a second analysis analogous to the widely used variational *outer loop*. If we also use the updated perturbation weights $\mathbf{W}^a_n$, the outer loop algorithm is extended into "running in place" (RIP).

Table 1 shows the result of optimized 4D-Var and LETKF applied to the Lorenz (1963) model (Kalnay et al., 2007a, Kalnay and Yang, 2009). 8-step windows are sufficiently short that ensemble perturbations remain linear, but with 25-steps windows the perturbations grow nonlinearly and become non-Gaussian. With a linear window the 4D-Var (with optimized background error covariance and using Pires et al (1996) approach to extend the optimal window), and LETKF (with 3 ensemble members and optimal inflation), give similar RMS analysis errors. With a nonlinear window of 25 steps, 4D-Var is clearly better than the optimal LETKF. However using a single iteration of either the outer loop or the running in place algorithm every analysis window, the LETKF becomes considerably more accurate than 4D-Var.

The RIP algorithm can also be used to accelerate the spin-up of an ensemble (Kalnay and Yang, 2009) so that in real time the LETKF converges to the optimal solution even if started from an ensemble without any prior information.

RMS analysis errors	4D-Var	LETKF	LETKF + outer loop	LETKF + RIP
Window=8 steps	0.31	0.30	0.27	0.27
Window=25 steps	0.53	0.66	0.48	0.39

Table 1: Comparison of the analysis RMS errors with optimized 4D-Var and optimized LETKF for a short window (8 steps) during which the ensemble perturbations remain linear, and a long window (25 steps), during which the perturbations grow nonlinearly and become non-Gaussian. One iteration of either the "outer loop" (re-centering the ensemble around the smoothed mean) or the RIP algorithm (smoothing both the ensemble mean and the perturbations) is enough to address this problem. From Yang and Kalnay, 2009.

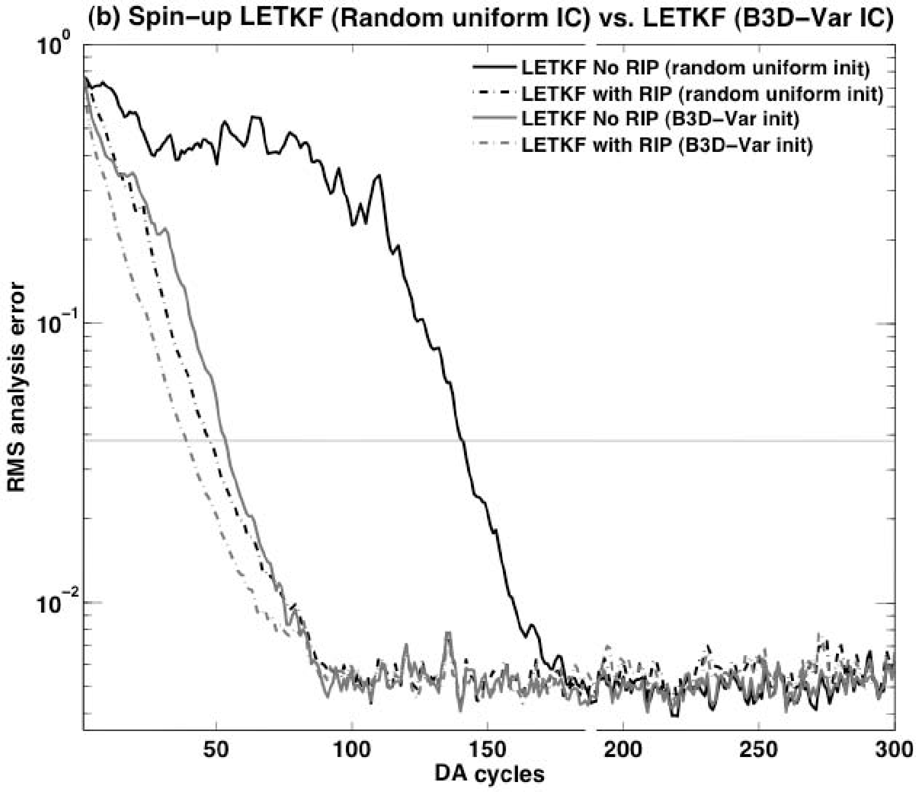

Figure 2: Comparison of the spin-up of an ensemble started from the same random mean, with and without RIP. The initial perturbations are random, with either a uniform distribution (black) or drawn from the 3D-Var background error covariance (grey). From Kalnay and Yang, 2009.

Figure 2 shows an example for the Rotunno and Bao (1996) QG model where the LETKF was started from a randomly chosen mean and from perturbations which were either random with a uniform distribution, or from the tuned 3D-Var background error covariance. The application of the RIP algorithm reduces substantially the spin-up time for both a very poor and the best available choice of initial ensemble members.

4. Coarse analysis with interpolated weights

Yang et al. (2008) compared performing analysis in coarse grids and interpolating the analysis fields or the analysis increments to a fine grid, with the interpolation of the perturbation weight matrices $\mathbf{W}^a$ and the analysis weights $\overline{\mathbf{w}}^a$.

Figure 3 shows that the accuracy of the analysis increments is quickly lost when interpolated increments (similar results are obtained when interpolating full fields). For the weight interpolation, the accuracy of the increments and their dynamical structure is maintained even when the analysis is performed every 7x7=49 grid points (a coverage of only 2%). This indication that interpolation of the weights more accurate is confirmed by the results shown in Figure 4. In fact the coarse resolution analysis with weight interpolation is slightly more accurate than the full resolution analysis. This indicates that the weights represent large scales and should be smoothed to avoid sampling errors in the smaller scales.

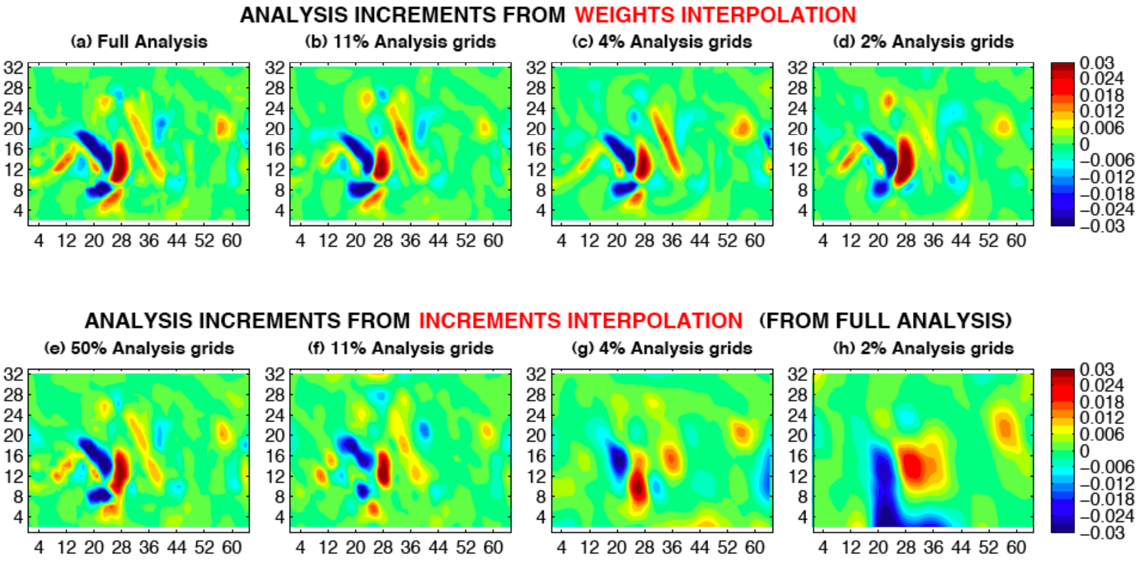

Figure 3: Top: comparison of the analysis increments for the full grid analysis resolution, and analyses performed every 3x3, 5x5 and 7x7 grid points, when the LETKF weights are interpolated. Bottom: analysis increments when the analysis increments are interpolated from the analysis at every other grid point, and 3x3, 5x5 and 7x7 grid points.

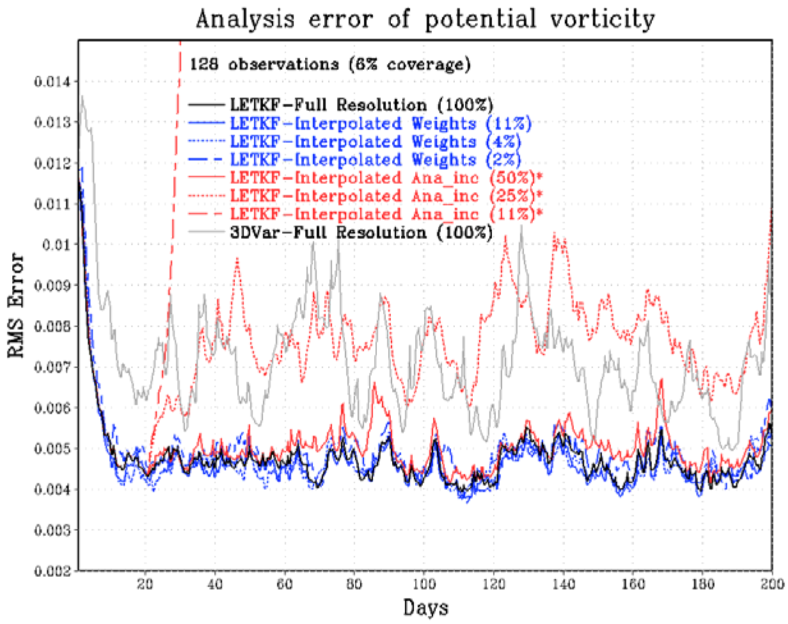

Figure 4: Analysis error for a QG model simulation, with 128 sounding observations every 12 hours. Black: LETKF full analysis resolution. Grey: 3D-Var full analysis resolution/ Red lines: LETKF at lower resolution interpolating the analysis increments. Blue lines: LETKF at lower resolution interpolating the weights. From Yang et al. (2009)

5. Handling model errors

Model errors should be accounted for in the formulation of EnKF as well as in 4D-Var. In the standard 4D-Var formulation, minimizing the cost function with the initial conditions of the model trajectory as control variables imposes a strong constraint, neglecting model errors. Only recently has the possibility of accounting for model errors by imposing the model trajectory as a weak rather than a strong constraint has been considered (Tremolet, 2006). Another approach is to estimate the model bias by repeated comparisons with the observations (e.g., Dee and Da Silva, 1998) so that the model bias is defined in observation space. The

Low-Dimensional method of Danforth et al. (2008) also estimates the bias, as well as the diurnal cycle and the state-dependent errors, but these correction fields are defined in model space.

In EnKF the most common approach to handling model errors has been to increase the background error covariance by a multiplicative factor greater than 1, or adding random numbers or fields to the covariance. These methods are known as multiplicative or additive inflation respectively, and of the two, additive inflation has been found to be slightly more effective, presumably because it forces the ensemble to explore wider subspaces (Whitaker et al., 2008). Another approach is the relaxation to the prior covariance of Zhang et al. (2004). Even for perfect model simulations, it is found necessary to use an inflation of the background error covariance of, typically, 1-10% to avoid filter divergence. For imperfect models, it has been empirically found that the required inflation is much larger than for perfect models, typically 20-100%, and that inflation increases with the observation density.

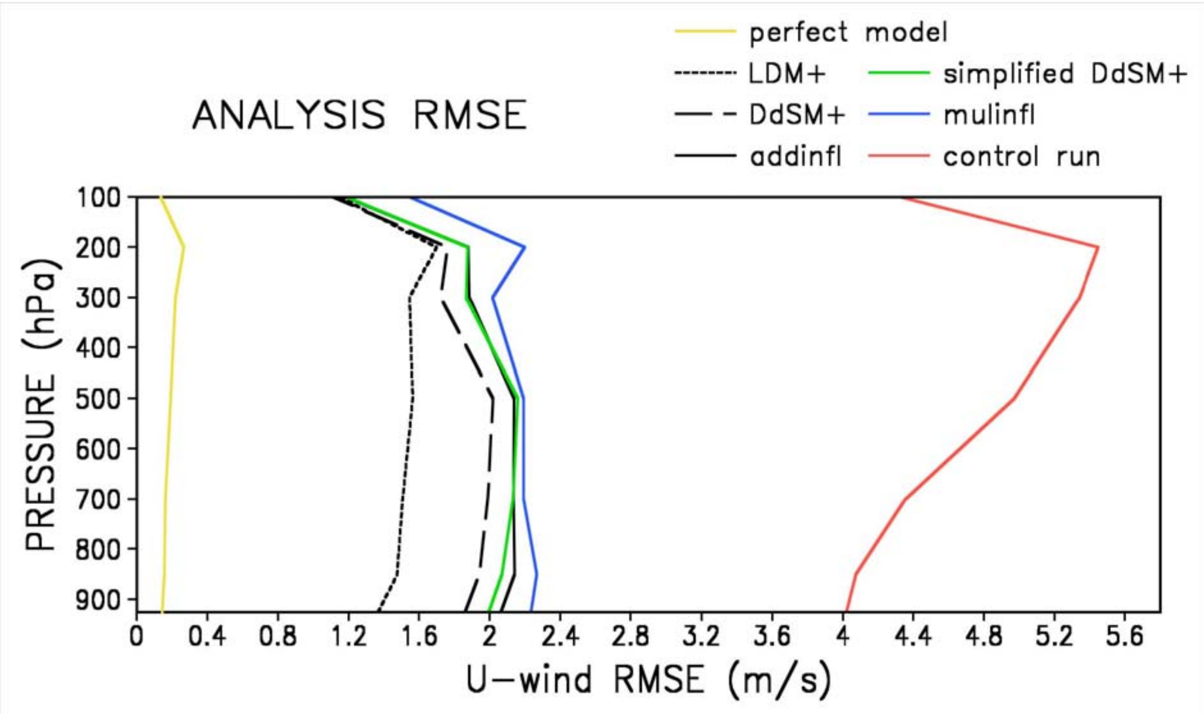

Figure 5: Comparison of the time average analysis error for the zonal velocity u for a perfect model (yellow) and for observations derived from the NCEP-NCAR Reanalysis using the same inflation found optimal for the perfect model (control, red). For each of the following methods, parameters were optimized: multiplicative inflation (blue), additive inflation (black). The three methods that estimate bias were found to give somewhat worse results than optimal inflation, so they were optimally combined with additive inflation (indicated by a +). Dee and DaSilva (black dashes), simplified Dee and DaSilva (green) and Low-Dimensional Method (black dots). From Li et al. (2009a).

Li et al. (2009a) made a carefully optimized comparison of several methods to handle model errors within the LETKF. The experiments were performed with the SPEEDY model (Molteni 2003), first assuming a perfect model experiment, with the observations obtained from a “nature run” adding random errors, and then observations extracted from the NCEP-NCAR Reanalysis, which approximately follows the real atmosphere rather than the SPEEDY model climatology. Several methods were implemented with optimized parameters, and their results were compared. They include additive and multiplicative inflation (with the former giving slightly better results than the latter); the Dee and DaSilva method, a simplified Dee and DaSilva method, and the Low-Dimensional method (LDM) adapted from Danforth et al. (2008). The last three methods estimate and correct the bias, whereas the additive and multiplicative inflation are naturally better suited for random errors. They found that inflation was better than bias correction alone, but that the optimized combination of bias correction and additive inflation was superior to either one alone (Figure 5). When using realistic observations, all these methods gave substantial improvements compared to the perfect model assumption (control run).

6. Simultaneous estimation of inflation and observation error covariance

Any statistical data assimilation method requires accurate estimations of the background and the observation error covariances. Sometimes, in the absence of good estimates these error covariances are just tuned or obtained from reasonable assumptions. In EnKF, the background error covariance needs to be inflated, and as previously mentioned, tuning the inflation can be an expensive effort.

Li et al. (2007, 2009b) introduced a method to simultaneously estimate observation error variances and inflation. Miyoshi et al. (2009) has recently extended to the estimation of correlated observation errors. These methods are inspired by the previous work of Houtekamer et al. (2001) who proposed to testing the validity of the statistical assumptions using an equation which we refer to by the operation "observation minus background" OMB:

$$<\mathbf{d}_{o-b}\mathbf{d}_{o-b}^T>=\mathbf{H}\mathbf{P}^b\mathbf{H}^T+\mathbf{R} \qquad \text{OMB*OMB}$$

This equation should be satisfied if the statistical assumptions are correct. However, with inflation, we have to multiply the background error covariance by a number $\Delta>1$:

$$<\mathbf{d}_{o-b}\mathbf{d}_{o-b}^T>=\mathbf{H}\Delta\mathbf{P}^b\mathbf{H}^T+\mathbf{R}$$

However, if we try to estimate inflation by assuming that the observation error covariance is accurate, when in fact they may be wrong by as much as a factor of 2, the results are poor. Desroziers et al. (2005) introduced two new statistical relationships that allow to handle uncertainties in both background and observation error covariances:

$$<\mathbf{d}_{o-a}\mathbf{d}_{o-b}^T>=\mathbf{R} \qquad \text{OMA*OMB, and}$$

$$<\mathbf{d}_{a-b}\mathbf{d}_{o-b}^T>=\mathbf{H}\mathbf{P}^b\mathbf{H}^T \qquad \text{AMB*OMB.}$$

Again, these three relationships should hold during the data assimilation if the background and observation error covariance are correctly estimated and the assumptions about uncorrelated errors are valid.

Like Desroziers et al. (2005), Li et al. (2007) transposed these matrix relationships and obtained three equations to estimate what the observation error variances and the inflation should be. These were used adaptively during the execution of the LETKF to estimate the optimal value of these parameters:

$$\Delta^o=\frac{(\mathbf{d}_{o-b}^T\mathbf{d}_{o-b})-Tr(\mathbf{R})}{Tr(\mathbf{H}\mathbf{P}^b\mathbf{H}^\mathbf{T})} \qquad \text{OMB*OMB}$$

$$\Delta^o=\sum_{j=1}^{p}(y_j^a-y_j^b)(y_j^o-y_j^b)/Tr(\mathbf{H}\mathbf{P}^b\mathbf{H}^T) \qquad \text{AMB*OMB}$$

$$(\tilde{\sigma}_o)^2=\mathbf{d}_{o-a}^T\mathbf{d}_{o-b}/p=\sum_{j=1}^{p}(y_j^o-y_j^a)(y_j^o-y_j^b)/p \qquad \text{OMA*OMB}$$

Li et al (2007) used a simple time smoothing scheme to estimate both the inflation Δ and the observation error variances σ_o^2 online. In the experiments with the SPEEDY model, the observation errors for the wind components, temperature, moisture and surface pressure were all wrongly specified, with a value twice the "true" value. It is interesting that the system initially ignored the inflation value (assumed $\Delta = 1$) and concentrated in correcting the observation errors, and only then started estimating the time varying inflation (Figures 6a and b).

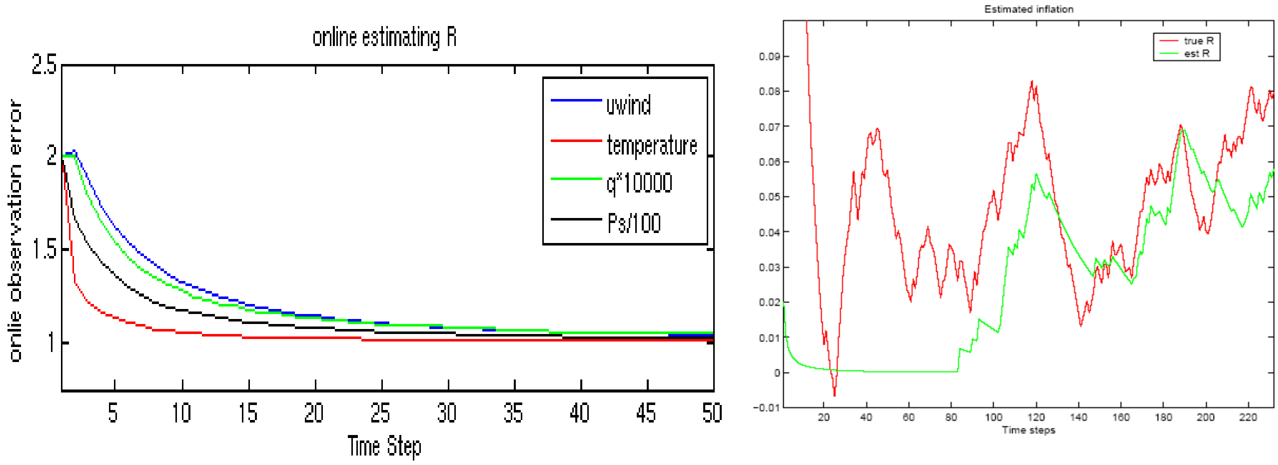

Figure 6a (left): evolution of the observation error variances starting with a value too high by a factor of 2. Figure 6b (right): evolution of the inflation when the observation error variances are correctly specified (red values, hovering around 1.005), and those obtained when the observation error variances are wrongly specified (green). Note that initially the system focused in correcting the variances, while keeping the inflation at 1, and only after the variances are corrected, the inflation is estimated at similar values as with perfect observational variances. From Li et al. (2009b).

Li et al. (2009b) obtained good results with adaptive inflation in the presence of random model errors, even for large errors. In the presence of model biases, the results were satisfactory for low bias errors but worked less well for high bias, indicating that inflation, which is best for random errors, needs to be combined with a method to account for model bias. Miyoshi (2009, in preparation) has tested with success the estimation of correlated observation errors, an especially promising approach for estimating the observation error covariance for satellite retrievals that may be accurate but whose correlated errors are unknown.

7. Ensemble forecast sensitivity to observations without adjoint

Langland and Baker (2004) developed an extremely powerful method to estimate the impact of observations on short-range forecasts. Liu and Kalnay (2008) derived a similar ensemble forecast sensitivity without adjoint. There is a slight error in the formula (Li et al., 2009c), so we re-derive the ensemble sensitivity here (Junjie Liu, pers. comm., 2009).

The perceived error from the forecast at t=0hr, verified against the analysis at time t, is $\mathbf{e}_{t|0} = \overline{\mathbf{x}}_{t|0}^f - \overline{\mathbf{x}}_t^a$, and from the forecast started at t=-6hr, $\mathbf{e}_{t|-6} = \overline{\mathbf{x}}_{t|-6}^f - \overline{\mathbf{x}}_t^a$. The only difference between $\mathbf{e}_{t|0}$ and $\mathbf{e}_{t|-6}$ is the assimilation of observations at t=00hr performed with the standard formula: $(\overline{\mathbf{x}}_0^a - \overline{\mathbf{x}}_{0|-6}^b) = \mathbf{K}(\mathbf{y} - H(\mathbf{x}_{0|-6}^b))$

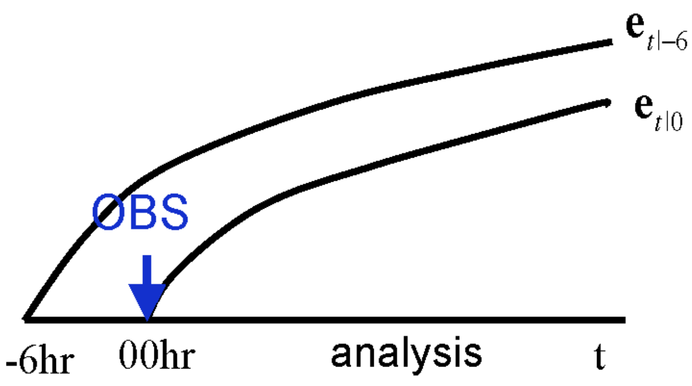

Figure 7: Schematic (adapted from Langland and Baker, 2004) showing how the difference between $\mathbf{e}_{t|0}$, the perceived error of the forecast started at t=0 verified at time t, and $\mathbf{e}_{t|-6}$, the perceived error of the forecast started at t=-6hr, both verified against the analysis at time t, is due to the assimilation of the observations at time t=0.

We measure the impact of the observations at 00hr from the difference of the square of the errors (note that we have used the Eulerian norm in this inner product it can be easily generalized to the energy or any other norm):

$$\begin{aligned}\Delta \mathbf{e}^2 &= (\mathbf{e}_{t|0}^T \mathbf{e}_{t|0} - \mathbf{e}_{t|-6}^T \mathbf{e}_{t|-6}) = (\mathbf{e}_{t|0}^T - \mathbf{e}_{t|-6}^T)(\mathbf{e}_{t|0} + \mathbf{e}_{t|-6}) \\ &= (\overline{\mathbf{x}}_{t|0}^f - \overline{\mathbf{x}}_{t|-6}^f)^T (\mathbf{e}_{t|0} + \mathbf{e}_{t|-6}) \\ &= \left[\mathbf{M}(\overline{\mathbf{x}}_0^a - \overline{\mathbf{x}}_{0|-6}^b)\right]^T (\mathbf{e}_{t|0} + \mathbf{e}_{t|-6})\end{aligned}$$

so that the formula for forecast sensitivity is

$$\Delta \mathbf{e}^2 = \left[\mathbf{MK}(\mathbf{y} - H(\mathbf{x}_{0|-6}^b))\right]^T (\mathbf{e}_{t|0} + \mathbf{e}_{t|-6})$$

Langland and Baker (2004) and Zhou Gelaro (2008) compute this impact using adjoint relationships, requiring the adjoint of the model $\mathbf{M}^T$ and of the data assimilation $\mathbf{K}^T$:

$$\Delta \mathbf{e}^2 = \left[(\mathbf{y} - H(\mathbf{x}_{0|-6}^b))\right]^T \mathbf{K}^T \mathbf{M}^T (\mathbf{e}_{t|0} + \mathbf{e}_{t|-6})$$

Adjoint forecast sensitivity (L&B, 2004)

Within EnKF we can use the original equation without the need to compute the adjoint. Recall that $\mathbf{K} = \mathbf{P}^a \mathbf{H}^T \mathbf{R}^{-1} = 1/(K-1)\mathbf{X}^a \mathbf{X}^{aT} \mathbf{H}^T \mathbf{R}^{-1}$ so that

$$\mathbf{MK} = \mathbf{MX}^a (\mathbf{X}^{aT} \mathbf{H}^T) \mathbf{R}^{-1} / (K-1) = \mathbf{X}_{t|0}^f \mathbf{Y}^{aT} \mathbf{R}^{-1} / (K-1).$$

Thus, for EnKF we can write directly

$$\Delta \mathbf{e}^2 = \left[(\mathbf{y} - H(\mathbf{x}_{0|-6}^b)\right]^T \mathbf{R}^{-1} \mathbf{Y}_0^a \mathbf{X}_{t|0}^{fT} (\mathbf{e}_{t|0} + \mathbf{e}_{t|-6}) / (K-1)$$

Ensemble forecast sensitivity

Note that this is a product using the nonlinear forecast ensemble $\mathbf{X}_{t|0}^{fT}$ and the ensemble analysis perturbations in observation space $\mathbf{Y}_0^a = (\mathbf{HX}^a)$, available within the EnKF formulation.

These formulas suggest that the adjoint and ensemble sensitivities should be similar for short forecasts, for which the linear assumption of the adjoint model is valid. For longer integrations, the ensemble sensitivity uses nonlinear integrations so that it should be more accurate. As in Liu and Kalnay, we tested both the adjoint and the ensemble sensitivities using the Lorenz (1996) 40-variable model, and performed an LETKF data assimilation using simulated observations.

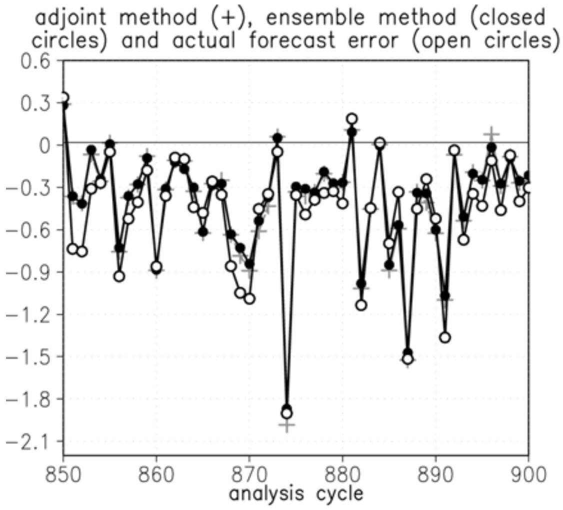

Figure 8a: Comparison of the day-to-day domain averaged changes in forecast error reduction due to observation random errors at t=00hr after a 1-day forecast with the Lorenz 40 variable model. The full black circles are obtained with the ensemble sensitivity, the grey + signs correspond to the adjoint sensitivity, and the open circles are the actual decrease in errors due to the observations at analysis time.

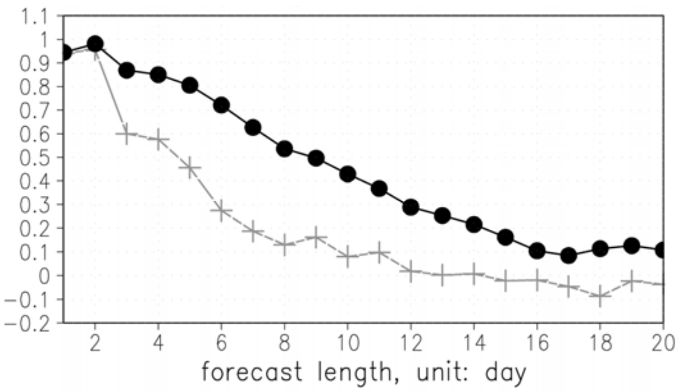

Figure 8b: time correlation between the predicted forecast error reductions and the actual forecast error reductions for different forecast lengths. Black circles: ensemble sensitivity, Grey +: adjoint sensitivity.

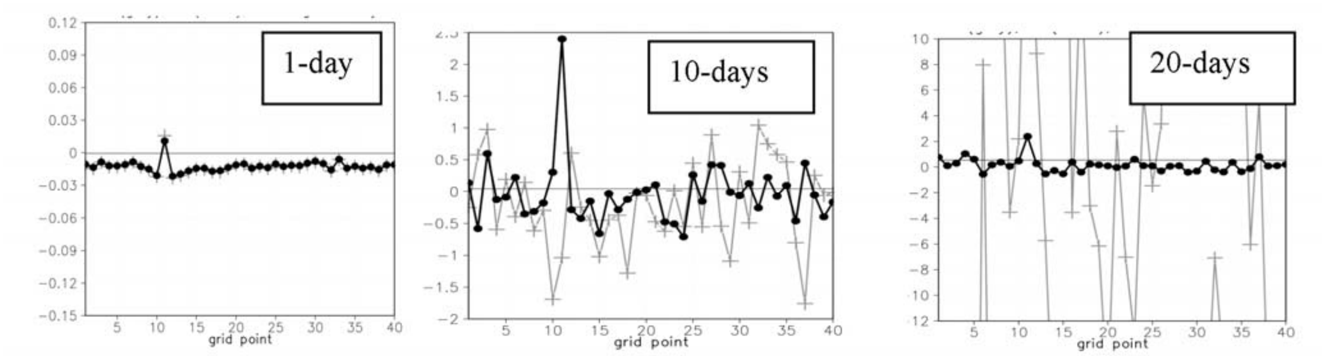

Figure 9: Time average of the forecast sensitivity to each of the 40 observations. The observation at location 11 is drawn from a Gaussian distribution with standard deviation double than the other stations, and this information is not provided to the system. Left: 1 day, center: 10-days, right; 20 days. Black circles: ensemble sensitivity, + signs: adjoint sensitivity.

Figures 8a compares the adjoint and ensemble estimated impact of the observations at each analysis time with the actual observed impact by computing the one-day forecast and the forecast started from 6 hours earlier. It is clear that both methods do very well, and that the time correlation due to day-to-day variations in observation errors is almost 1. Figure 8b shows the time correlation for different forecast lengths, up to 20 days, a time at which the forecast skill is essentially zero. The advantage of using nonlinear integrations instead of the linear adjoint model is apparent. In Figure 9 we have observations at every grid point, but at grid point 11 there is a “bad sensor” with random errors that have twice the standard deviation of the other stations. As expected, after 1-day, the adjoint and ensemble sensitvity give essentially identical results. After 10 days the ensemble sensitivity is still able to identify the bad observations, and that seems to hold even at day 20, when the forecasting system has very little skill left.

8. The future

EnKF has benefited from the many years of 4D-Var research and operational experience. We have shown that methods derived to improve 4D-Var can be adapted to EnKF. The main disadvantage of EnKF is the fact that the analysis increments are computed within the subspace of the ensemble, of dimension *K*, seemingly too small. However, the introduction of localization in space, needed to avoid spurious long distance correlations (Hotekamer et al., 2001), has the benefit of substantially increasing the effective dimension of the space spanned by the ensemble. This seems to be confirmed by the fact that good results have been obtained with ensembles of size 40-100, and not much additional advantage observaed from further increases of the ensemble size. Even fewer ensembles are needed for less chaotic systems than the Earth’s atmosphere (Matt Hoffman, personal communication, 2009).

EnKF is particularly good at estimating evolving parameters within the analysis cycle. Adaptive localizations have been proposed by Bishop and Hodyss (2009) and by Anderson (2007). Recently Miyoshi (personal communication, 2009) has tested with good results an adaptive localization scheme that takes advantage of both these methods. This adaptivity and the ability to estimate observation errors is helpful for long reanalyses, when the observing systems (and their associated errors) keep changing.

At this point it seems like a hybrid system combing the advantages of both 4D-Var and EnKF is optimal, although there are several possible ways to construct a hybrid system (Barker, 2008, Buehner, 2008), so that we expect to see both 4D-Var and EnKF thriving for many more years.

9. Acknowledgements

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