# TEST OF A SPECTRAL LIMITED AREA SHALLOW WATER MODEL WITH TIME-DEPENDENT LATERAL BOUNDARY CONDITIONS AND COMBINED NORMAL MODE/SEMI-LAGRANGIAN TIME INTEGRATION SCHEMES

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## 1. INTRODUCTION

Within the HIRLAM (High Resolution Limited Area Modelling) project, an ongoing 3 year joint research and development project among the Nordic countries and the Netherlands, a series of experiments with alternative spectral representations and various time integration schemes have been carried out. In the present paper the main results of these experiments will be summarized. For a detailed description the reader is referred to HIRLAM Technical Report No. 4, which will be available before the end of the project (August 1988).

Most of the currently used medium range global models utilize the spectral method for horizontal discretization.

For presently obtainable resolutions at least, global spectral models seem to be more efficient as regards computer speeds and memory requirements than grid point models. Due to a high order of accuracy in the computation of derivatives, less degrees of freedom are required to achieve the same degree of accuracy. In particular linear advection is computed exactly except for time truncation and round off errors and thus no computational dispersion due to space discretization is introduced. Principal non-linear terms are computed as a least square fit, without any aliasing and consequently a source of non-linear instability is eliminated. By the use of representations in terms of surface spherical harmonics no pole-problem is encountered and with a triangular truncation a homogeneous resolution over the globe is achieved. As the expansion functions are eigenfunctions to the Laplace operator, significant computational advantages are achieved in the computations connected with horizontal diffusion, semi-implicit time extrapolation and non-linear normal mode initialization schemes.

The over all efficiency of the spectral method relies, however, heavily on the efficiency of the transformations from spectral to grid point space and vice versa, which are performed each time step. The Fourier transformations in the east-west direction are speeded up considerably by the use of FFT routines. A similar "fast Legendre transform" to be used for the north-south transformations has, however, not been found, which implies an increasing dominance of the computational cost of the Legendre transforms with increasing resolution. The efficiency of the spectral method, relative to finite difference and finite element methods, therefore decreases with increasing resolution. With present formulations of global medium range forecast models an advantage in efficiency of the spectral method probably still exists even for the highest resolutions (T 106) used operationally.

The transform grid, in physical space, used in global models is a lat-long grid, equidistant in longitude and very nearly equidistant in latitude, when using a triangular truncation. It should be possible, without significant loss of accuracy, to decrease the number of points along the latitude circles as the poles are approached, whereby the resolution in the transform grid are brought closer to the homogeneous resolution implied by a triangular spectral representation (Machenhauer 1988). Such a formulation, which would improve the efficiency, has, however, not yet been developed and tested in practice.

Due to the over all advantages of the spectral method we decided at the start of the HIRLAM-project to investigate the possibilities for its application to high resolution regional models.

One possibility, initially investigated, was, as proposed by Schmidt (1977), to introduce a variable resolution in a global spectral model. This could be achieved relatively easy by modest modifications to the model equations, i.e. by the introduction of mapping factors in certain terms.

An alternative was to try to develop a spectral limited area model with prescribed time dependent lateral boundary values obtained from a global model, similar except for the horizontal discretization to the finite difference LAM that were under parallel development and tests within the HIRLAM-project (Machenhauer (1988)).

Experiments aiming at such a spectral LAM had already been started up at ECMWF by Simmons (1984) and shortly after the start of the HIRLAM-project we learned that a similar development had taken place at the Japanese Meteorological Agency, JMA, where Tatsumi (1985) had carried out tests with very promising results.

## 2. BACKGROUND OF THE CHOICE BETWEEN THE TWO ALTERNATIVE APPROACHES.

The approach suggested by Schmidt (loc. cit.) must be very attractive for a centre which wants to run operationally a medium range prediction system as well as a regional short range prediction system. It seems possible that the same model in relatively similar versions, can be used for the two purposes. Thus, if this turns out to be the case the maintenance and development of the combined system would be easier than for two separate systems. The experimentation and further development of the Schmidt approach undertaken by Courtier and Geleyn (1987) is therefore certainly relevant.

In the weather services participating in the HIRLAM-project, however, the plans are to develop a short range regional forecasting system only and to rely on the ECMWF products for medium range forecasting. Taking into account this situation and the uncertainties as to whether it would be possible to develop a spectral regional model that in all respects would be competitive with more traditional formulations we decided to put an effort primarily into the development of an up-to-date grid point model.

In parallel with this development we decided to make experiments with a spectral formulation which at a later stage could be utilized with a minimum of changes to the primary grid point system.

In the design and coding of the grid point model we have taken into account this possibility and we have aimed our development of a spectral formulation towards one that utilizes a transform grid similar to the grid used in the grid point model.

In the grid point model the integration area is a rectangle with sides parallel to the coordinate axes in a rotated spherical coordinate system. In a spectral LAM with such a transform grid it seemed possible that the same expansion functions could be applied in the two coordinate directions. If a double Fourier representation could be used, an advantage over global models, and thus over Schmidt's approach, would be that FFT's could be used in both coordinate directions.

An additional advantage would be that, unlike global spectral models the ratio between the resolution in the grid and in the spectral representation would be homogeneous over the integration area.

Finally, if we had chosen the Schmidt approach we would have had to use a variable resolution over the area of interest. Although this may ultimately turn out to be an advantage several problems may be anticipated. They can probably be solved, but it will be an additional task which will require an unpredictable amount of development work. With the approach we have chosen, it is possible to have an almost homogeneous resolution over the integration area, at least initially, and then later on to introduce a variable resolution, if this turns out to be desirable.

## 3. BACKGROUND OF THE NEW SPECTRAL FORMULATION.

When trying to formulate a spectral one-way interacting limited area model, which should build upon a representation in terms of double Fourier expansions, the main problem encountered is how to formulate the boundary conditions. Usually the externally determined boundary values are imposed in a boundary zone by a relaxation scheme. This procedure is used in the spectral LAM's developed at ECMWF and JMA and will be used in our formulation as well.

The problem is, of course, that when Fourier series are to be used a cyclic domain is assumed. As the boundary relaxed values as well as their slopes are in general different at opposite boundary points, discontinuities in both quantities are introduced when the domain is made cyclic. A Fourier series representation can be used in spite of such discontinuities, but in general problems with very slow convergence of the series and large Gibbs waves in the resulting representation will be experienced in the neighbourhood of the discontinuites, i.e. at the boundaries of the integration area.

In the scheme proposed by Simmons (loc. cit.) a boundary relaxation scheme is applied at the end of each time step. This implies that the deviations of the variables from the corresponding background fields, toward which the fields are relaxed in the boundary zone, become zero at the boundary. Thus, such deviation fields have no discontinuities of zero order at the boundaries. Simmons' initial suggestion was to represent only such deviations spectrally and to use representations in terms of truncated double sine series. The prediction equations used for the deviations involve not only the deviation fields but also the background fields as well as the time derivatives of these fields. For efficiency reasons at least, the appearance of the background field and its space and time derivatives in the prediction equations is a disadvantage, as these additional quantities must be computed and used over the whole integration area each time step. Furthermore a basic property of the spectral method is lost as the time derivatives of the deviation fields due to non-linear interactions do not become truncated series of the expansion functions. Thus the transform method does not ensure least square fit and aliasing free truncations.

Some of the drawbacks of the formulation suggested (initially) by Simmons is less pronounced in the spectral formulation proposed and tested by Tatsumi (1985, 1986). He is using representations of the variables which in each direction are either a sine or a cosine series. The quantities represented is deviations from certain large scale fields. These large scale fields (represented by Fourier components with wave length of the order of, or larger than, the dimensions of the integration area) are determined from the boundary values. The representations of the deviation fields satisfy solid wall boundary conditions and the large scale fields are added in order to specify quantities (values of variables and their derivatives normal to the boundaries) that are not determined by the solid wall boundary conditions. As in Simmons' formulation the prediction equations for the deviation fields involve, beside deviation fields, also the large scale fields and their time derivatives. Thus also in the Tatsumi formulation externally determined fields are needed in the whole of integration area but these are more easily determined than the full background field quantities that are involved in the formulation suggested by Simmons. Like Simmons' method Tatsumi's formulation gives no guarantee of least square fit and non-aliased truncations. However, it seems likely that these properties are maintained to a larger extent in Tatsumi's formulation than in that of Simmons'.

It should be mentioned that certain changes have been made to the formulation suggested initially by Simmons, so that in particular the choice of basis-functions used at present at ECMWF corresponds more to the Tatsumi formulation.

In the following section we shall briefly present an alternative formulation. In this new formulation we avoid the inclusion inside the integration area of quantities determined from the background field, except, of course, in the relaxation zone. The formulation implies a representation of the variables by full Fourier series and not just sine or cosine series. A model based upon this formulation becomes less complicated and probably more efficient than the previously proposed formulations, referred to above.

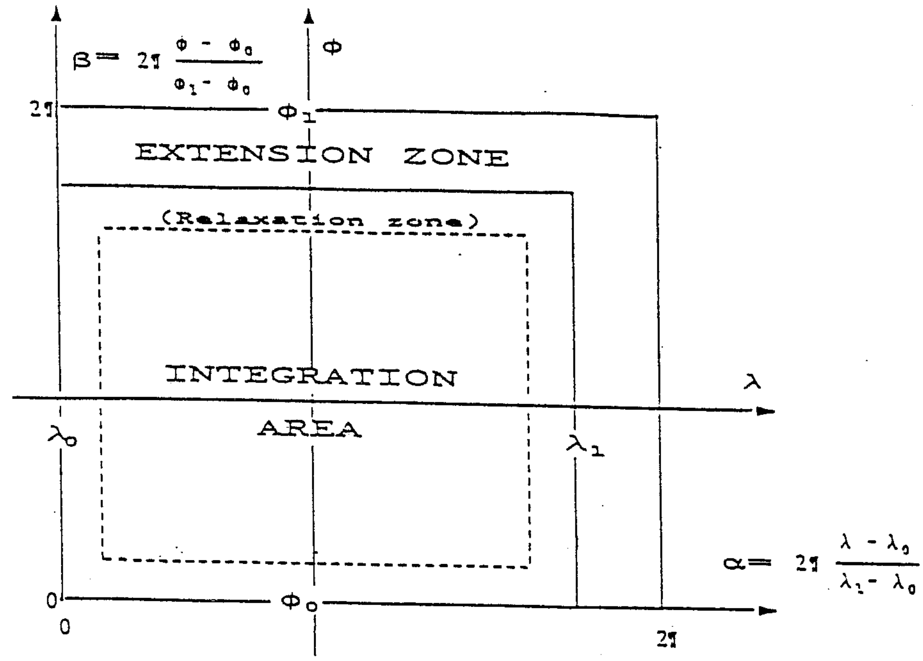


Fig. 1 Sketch of the integration area including boundary relaxation zone and the extension zone added to the integration area.

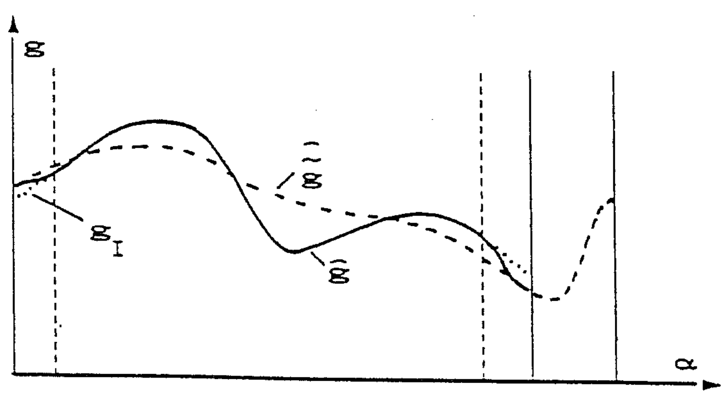


Fig. 2 Sketch of the spectral representation of a variable g in a cross-section along the $\lambda$-axis.


Fig. 3 Sketch of the relaxation coefficient $\gamma$ in a cross--section along the $\lambda$-axis.

## 4. THE NEW SPECTRAL REPRESENTATION

As illustrated in Figure 1 we use an integration area which is rectangular in a rotated spherical coordinate system with coordinates $\lambda$ and $\phi$. The choice of coordinate system is, however, not essential. Along the boundaries inside the integration area we define a relaxation zone and outside the integration area along the northern and eastern boundaries we define an extension zone. We introduce the coordinates $\alpha$ and $\beta$, defined in Figure 1, which both run from 0 to $2\pi$ over the extended area.

We want to represent a certain variable $g(\alpha, \beta, t)$ by a truncated double Fourier series of the form

$$\hat{g}(\alpha,\beta,t)= \frac{g_o^c(\beta,t)}{2} + \sum_{m=1}^{M} \left( g_m^c(\beta,t) \cos m\alpha + g_m^s(\beta,t) \sin m\alpha \right) \qquad (1)$$

where

$$g_m^c(\beta,t)= \frac{g_{m,o}^{cc}(t)}{2} + \sum_{n=1}^{N} \left( g_{m,n}^{cc}(t) \cos n\beta + g_{m,n}^{cs}(t) \sin n\beta \right)$$

for m= 0,1,.......,M

$$g_m^s(\beta,t)= \frac{g_{m,o}^{sc}(t)}{2} + \sum_{n=1}^{N} \left( g_{m,n}^{sc}(t) \cos n\beta + g_{m,n}^{ss}(t) \sin n\beta \right)$$

for m= 1,2,.......,M

In order to determine the expansion coefficients in these series by a usual discrete Fourier transform (FFT) we need grid point values $g_{i,j}$ over the whole extended area. These are obtained from grid point values $(g_I)_{i,j}$ evaluated in the transform grid inside the integration area and from grid point values $\hat{\tilde{g}}_{i,j}$ evaluated from the corresponding background field in the relaxation zone and in the extension zone. The two fields are fitted together using the relaxation scheme:

$$g_{i,j} = (1-\gamma_{i,j})(g_I)_{i,j} + \gamma_{i,j}\, \hat{\tilde{g}}_{i,j}$$

Where $\gamma_{i,j} = 1$ in the extension zone and decreases to 0 over the relaxation zone (see Figure 3).

The grid point values $\hat{g}_{i,j}$ are obtained from a truncated Fourier series representation of the form (1) which in turn is determined by a discrete Fourier transform using some externally determined grid point values $\tilde{g}_{i,j}$ as input. The background field g will typically be a field determined from a large scale forecast. We suppose it is given as a grid point field in the transform grid over the integration area. The basic idea utilized in the present procedure is to extend this grid point field into the extension zone in such a way that a smooth transition is obtained from the values at one boundary to those at the opposite boundary.

The extension of the $\tilde{g}_{i,j}$ field is done as follows:

From $\tilde{g}_{i,j}$ we determine at first, at all boundary points, the slope normal to the boundary. Next we use these slopes and the boundary values of $\tilde{g}$ to determine continuous functions in the extension zone which smoothly connects opposite boundary values of $\tilde{g}$. These functions are then used to determine grid point values $\tilde{g}_{i,j}$ in the extension zone.

The representation $\hat{g}$ obtained by a discrete Fourier transform of this extended grid point field $\tilde{g}_{i,j}$ is a least square fit with equal weight given to all grid points in the extended area. The extension of the grid point field $\tilde{g}_{i,j}$ serves to create a smooth transition between values at opposite boundaries and thereby to avoid Gibbs waves within the integration area. By the procedure chosen we try to get a smooth transition of values as well as of the derivatives normal to the boundaries.

The broader we make the extension zone the smoother does the transition across boundaries become and the less will the representation $\hat{g}$ inside the integration area be influenced by the auxiliary values introduced in the extension zone. With a sufficiently broad extension zone $\hat{g}$ will fit closely the given grid point values $\tilde{g}_{i,j}$ inside the integration area and at the boundary points, assuming, of course, that $\tilde{g}_{i,j}$ is sufficiently large scale to be representated well with the truncation used. In experiments reported on in the following we have used a width of the extension zone equal to 15-20 % of the sides of the integration area. In Figure 2 is shown an idealized sketch of the final representation g, in an "east-west" cross section, obtained for a variable g. Note, that only $\hat{g}$ is used in the prediction equations and that $\tilde{g}$ is used only as a field that is relaxed to in the boundary and extension zones.

## 5. SUMMARY OF TEST RESULTS.

### 5.1 Model and initial data sets.

The representation described above has been tested in a shallow water model. Model equations on a form similar to those used by Tatsumi (loc. cit.) have been used, except for modifications due to our choice of coordinate system. Tatsumi uses a Cartesian coordinate system on an arbitrary conformal map projection, whereas we have used rotated spherical coordinate systems.

We have made tests with the usual "lat-long" spherical coordinate system as well as with a new system which gives a more uniform resolution over the integration area. This new coordinate system, suggested by one of the authors, J. E. Haugen, will be described in the coming HIRLAM Technical Report No. 4.

Two sets of initial data were used in the tests performed.

The first set was a large scale Rossby mode obtained as an eigensolution to a linearized hemispheric spectral model. Two versions were used:

1) the uninitialized fields

and 2) the initialized fields obtained by using the non-linear normal mode initialization (NNMI) scheme of Machenhauer (1977).

The other initial data set was a "real" data set obtained from a HIRLAM baseline 500 hPa analysis.

### 5.2 Experiments with Rossby mode initial data.

Several experiments were made with the Rossby mode initial data. In all of these experiments boundary (or background) fields were forecast fields obtained with a semi-implicit R51 Hemispheric spectral shallow water model, given every 6th hour up to 36 hours after the initial time.

The limited area used was approximately 4500 × 4500 $km^2$ and centred at a point at 50° N. In the LAM a truncation approximately equal to that used in the Hemispheric model was used. Thus, we could test the formulations used in the LAM since its results should be more or less equal to those of the hemispheric model.

a) Experiments using the leap-frog scheme:

Experiments with the leap-frog scheme showed clearly that when large scale gravity oscillations were present in the initial field and in the subsequent hemispheric forecasts used as boundary fields, significant deviations between the LAM and the hemispheric forecast were experienced, whereas when such oscillations were eliminated by a hemispheric NNMI of the initial fields almost perfect LAM forecasts were obtained.

b) Tests of a LAM NNMI scheme:

The initialized hemispheric fields were then used to test a NNMI scheme developed for the LAM. The normal modes used in this scheme were obtained assuming a basic state at rest and mapfactors as well as the Coriolis parameter set equal to constants. The linearized forecast equations were assumed to be valid over the whole LAM area, including the extension zone. As the normal modes, with these assumptions, have horizontal structures equal to the Fourier components used in the expansion transformations between normal mode coefficients and Fourier coefficients become extremely easy.

In order to get the scheme to work properly it was found necessary to use background fields that were geostrophically balanced. With such background fields the projection on the gravity modes of the tendencies in the extension zone become zero. If unmodified background fields were used large ageostrophic tendencies in the extension zone lead to spurious initialization increments also in the adjacent integration area.

With this modification the LAM NNMI scheme seemed to work properly. As should be the case it gave only small increments to the hemispheric initialized fields within the LAM integration area. Futhermore, almost identical forecasts were obtained with and without the LAM NNMI applied to the initial data.

c) Experiments with a normal mode time extrapolation scheme:

As an alternative to a semi-implicite time extrapolation scheme experiments were then made with the normal mode initialization time scheme introduced by Daley (1980).

In this scheme the fields determined by the leap-frog scheme at time level $(n+1)\,\Delta t$ are corrected by substituting gravity modes balanced at time level $n\,\Delta t$ instead of the gravity modes determined initially by the leap-frog time step. The balanced gravity modes are obtained by one iteration step with the LAM NNMI scheme, using as input the tendencies at time level n $\Delta t$, which were used in the initial leap-frog time step.

This normal mode time stepping scheme was tested and found to work satisfactory for the Rossby mode initial data with a time step $\Delta t = 900s$, nine times the $\Delta t$ used in the leap-frog scheme. As should be expected the results were even closer to the semi-implicit hemispheric forecast than those of the LAM forecast with the leap-frog scheme.

## 5.3 Experiments with real data.

a) Integrations with leap-frog and with normal mode time extrapolation schemes:

In the experiments described in the following we used as background fields some HIRLAM baseline 500 hPa analyses from the period 00UT 5 Sept. to 00UT 6 Sept. 1985, given with a 6 hour time interval. (The height fields are shown in Figure 3.1 in Gustafsson et al. (1986)).

We used an area of integration approximately equal to that used in the HIRLAM baseline experiments (Gustafsson et al. (1986)), i.e. an area of about $4500 \times 3600$ km$^2$. In a rotated, spherical coordinate system a resolution of $\Delta\lambda = \Delta\phi = 0.5°$ was used for the transform grid. This gives $82 \times 65$ grid points in the integration area and we added an extension zone with 14 and 15 grid points at the eastern and northern boundaries, respectively. The spectral truncation chosen was $M = 29$, $N = 24$ for the prognostic variables.

In all experiments the background fields used during the integrations were obtained by linear interpolation in time between the fields determined from the 500 hPa analysis fields, given each 6 hour.

At first an integration with the leap-frog time integration scheme was made using the background field from 00 UT 5 September as initial state. A very noisy forecast was obtained. When the initial field was initialized using our LAM NNMI scheme an initially smooth forecast was obtained, but noise was found to propagate with the speed of external gravity waves from the boundaries towards the centre of the integration area. When then the same initial data (and background fields) were used in an integration with the normal mode time stepping scheme, smooth forecast fields were obtained over the whole integration area.

The initial height field and that after 1 hour obtained in

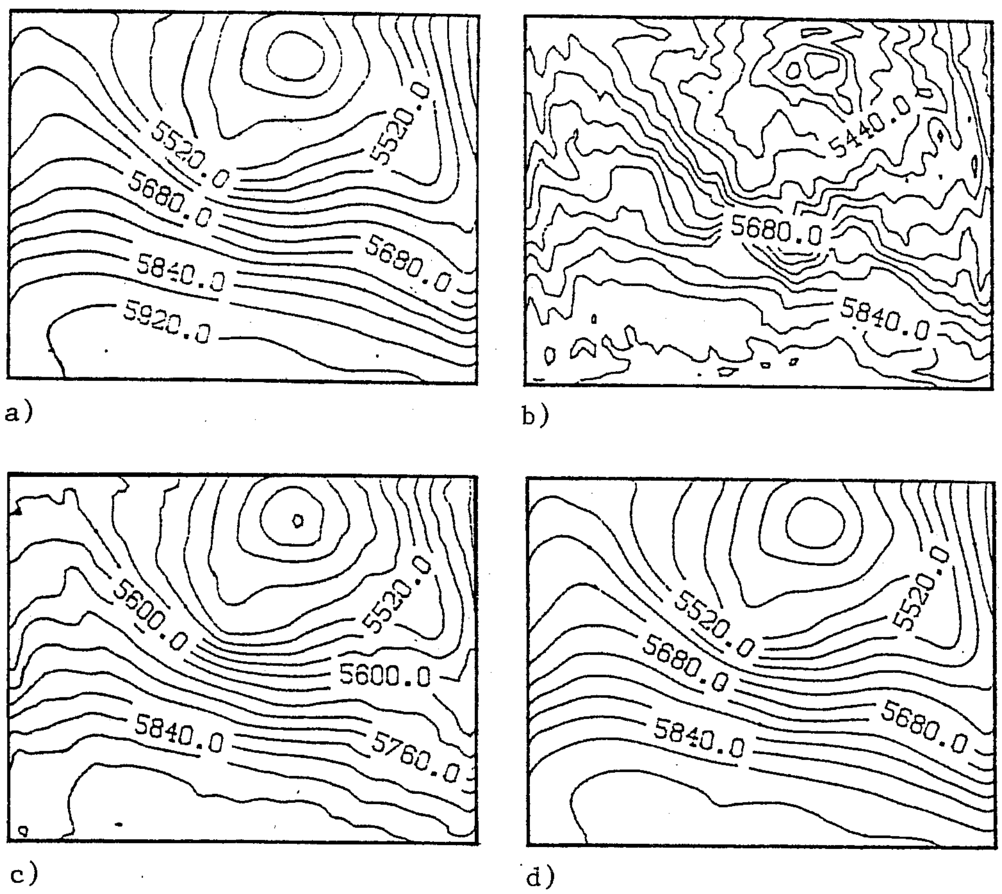


Fig. 4 a) HIRLAM baseline 500 hPa height analysis from 00 UT 5 Sept. 1985, initialized using the spectral LAM NNMI scheme.

b) 1 hour forecast with leap-frog time stepping from the uninitialized analysis (not shown).

c) 1 hour forecast with leap-frog time stepping from the initialized analysis (a).

d) 1 hour forecast with normal mode time stepping from the initialized analysis (a).

these three integrations are shown in Figure 4.

These experiments clearly demonstrated that it is important to have well balanced initial and boundary fields. The last of the three integrations demonstrated that the normal mode time extrapolation scheme effectively removes the noise created at the boundaries, due to unbalanced boundary fields.

In Figure 5a is shown the 24 hour forecast obtained with the normal mode time scheme. With this scheme a time step $\Delta t = 600s$ was used.

b) Experiments with combined semi-Lagrangian/normal mode time extrapolation schemes:

Before a decision could be taken on whether a spectral alternative to the HIRLAM multi-level grid point model should be developed, we found it necessary to test semi-Lagrangian methods in the spectral shallow water model. If, for some reason, such methods could not be used efficiently in the spectral model, this would be a great disadvantage compared to grid point models.

We have tested four different semi-Lagrangian methods using the same area, resolution and initial/boundary data as in a). The three schemes are:

1) The Geleyn scheme (personal communication)

2) The Ritchie (1986) scheme with bilinear, biquadratic and bicubic interpolation at the mid-point of the trajectory.

3) A new scheme suggested by Machenhauer. The formulation is as in the Ritchie scheme, but the departure point is taken as an even number of grid lengths from the arrival point in both horizontal directions. Such a scheme would not be stable in a grid point model, but in the spectral model it can be shown to be absolutely stable for linear advection if a certain, slightly more severe, truncation of the spectral fields is used. The advantage is that the mid point of the trajectory becomes a grid point, so that no interpolation is necessary. The overhead in required CPU-time is thereby minimized.

4) The Robert (1982) scheme with different interpolations (as for the Ritchie scheme) at the departure and at the mid point.

In all semi-Lagrangian experiments we had to reduce the integration area. This was done by changing the boundary

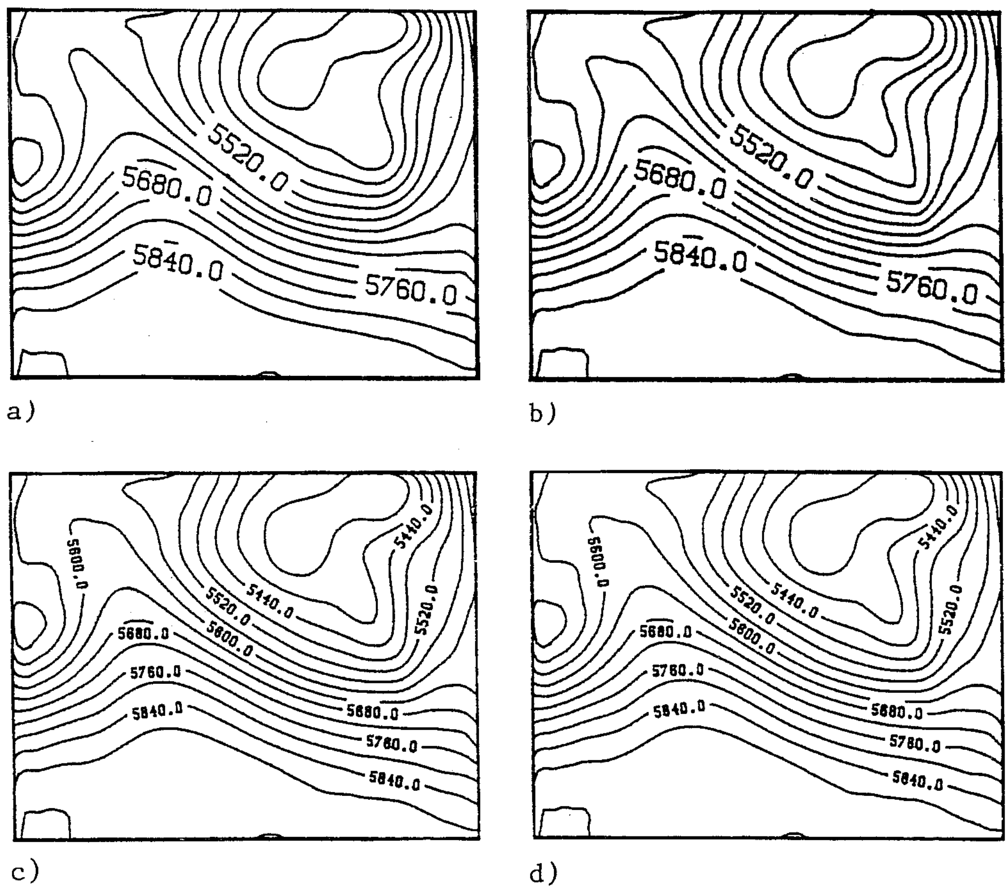


Fig. 5: 24 hour forecast from the initialized 500 hPa analysis valid 00 UT 5 Sept. 1985 (see Fig. 4a).

a) Eulerian integration on full area, $\Delta t$=10 min.

b) Eulerian integration on reduced area, $\Delta t$=10 min.

c) Semi-Lagrangian integration, Robert method, $\Delta t$=60 min.

d) Semi-Lagrangian integration, Machenhauer method, $\Delta t$=60 min.

relaxation scheme so that the outermost 8 grid point values after each time step were relaxed completely to the background values. This transition zone between the integration area and the extension zone was introduced in order to ensure that no trajectory had its departure point in the extension zone. The transition zone had to be taken from the original integration area as the background fields were only available on this area.

After some experimentation we found that it was necessary to compute the balanced gravity modes to be used in the normal mode time scheme from Eulerian tendencies. When we computed them from the preliminary semi-Lagrangian explicit forecast, instability of the shortest resolvable wavelengths occurred.

Compared to a usual normal mode time step, some additional spectral transforms are needed in all four semi-Lagrangian schemes.

Stable integrations with a time step of 3600s were performed with all the semi-Lagrangian schemes. When bicubic interpolations were used in the Robert and in the Ritchie schemes very similar results were found for all schemes. We had expected to see more damping in the Geleyn scheme than in the other schemes.

An Eulerian integration (Fig. 5b) over the reduced area showed systematic deviations from the Eulerian integration made over the full area (Fig. 5a). Basically these deviations implied a sharpening of a main ridge, which is entering the integration area from the west, as well as a sharpening of the main trough when it approaches the eastern boundary of the integration area. The Lagrangian integrations (Fig. 5c and 5d) showed the same deviations, but significantly less pronounced. Thus, semi-Lagrangian integrations compared more favourably with the Eulerian integration on the full area, which must be assumed to be the most "correct" integration. This may be due to a more correct treatment of the advection near the boundaries in the Lagragian integrations.

In Figure 5 is shown only two of the semi-Lagrangian forecasts, one using the Robert scheme (5c) and one using the Machenhauer scheme (5d). The time step used in the semi-Lagrangian forecasts were $\Delta t = 3600s$, six times that used in the Eulerian forecast.

In all integrations shown we have used the "elliptic" spectral truncation defined by $(N/M)^2 m^2 + (N/M)^2 n^2 \leqq N^2$, which gives an isotropic and homogeneous resolution over the extended area.

## 6. CONCLUDING REMARKS

Our conclusions from the shallow water experiments reported above is that the formulations tested seem to work properly and that they justify the development of a full multi-level model based on these formulations. Such a development is necessary in order to make realistic comparison with the HIRLAM grid point model. Unfortunately, this will not be possible within the present HIRLAM-project. However, such a development is recommended to be taken up in a planned continuation of the present HIRLAM-project.

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