Coupling the continuity equation with the physics in the IFS (Work in slow progress)

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- new continuity equation for conservation of dry mass instead of total mass
- net mass transport in mass flux scheme

Mass conservation

Lagrangian/Eulerian mass conservation

$$\frac{Dm}{Dt} = 0 \text{ or } \frac{\partial m}{\partial t} = -\vec{u}.\vec{\nabla}(m)$$

Equation for density

$$m = \rho V \Longrightarrow \frac{D\rho}{Dt} = -\rho \frac{1}{V} \frac{DV}{Dt} = -\rho D_3$$

What if an "unresolved" source of mass?

$$\frac{Dm}{Dt} = S \Longrightarrow \frac{D\rho}{Dt} = -\rho \frac{1}{V} \frac{DV}{Dt} + \frac{1}{V} S$$

- in an hydrostatic model, S need to be in hydrostatic balance,
- in a compressible model, S has to be understood by the model as a source of mass and not as a source of volume.

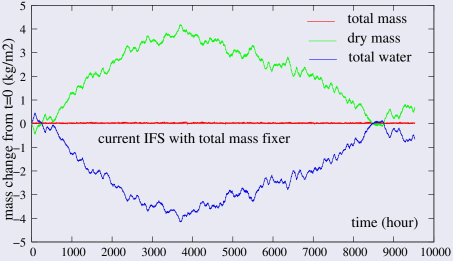
Total mass conservation versus dry mass conservation

Current IFS

$$\frac{D\rho_t}{Dt} = -\rho_t D_3$$

$$\frac{D\rho_w}{Dt} = -\rho_w D_3 - \mathcal{S}_\varphi$$

$$\Rightarrow \quad \frac{D\rho_d}{Dt} = -\rho_d D_3 + \mathcal{S}_\varphi$$

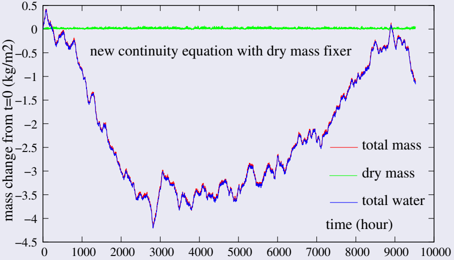


New continuity

$$\frac{D\rho_t}{Dt} = -\rho_t D_3 - \mathcal{S}_\varphi$$

$$\frac{D\rho_w}{Dt} = -\rho_w D_3 - \mathcal{S}_\varphi$$

$$\Rightarrow \quad \frac{D\rho_d}{Dt} = -\rho_d D_3$$



Total mass conservation versus dry mass conservation

In practice:

Current IFS

In the hydrostatic IFS, thanks to the hybrid pressure levels, the continuity equation is reduced to a 2D equation for the surface hydrostatic pressure:

$$\frac{\partial \pi_s}{\partial t} = \left[- \int_{surf}^{top} \vec{\nabla}.(\vec{v}\frac{\partial p}{\partial \eta})d\eta \right]$$

New continuity

Add physics tendency to the equation for the surface pressure, but also in the diagnostic equations for ω and $\dot{\eta}$ (the vertical velocity used for vertical advection).

Should the physics mass tendency affect all specific variables?

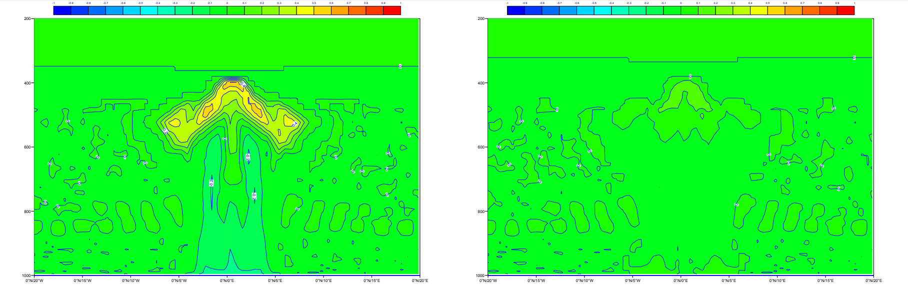
$$\frac{\partial(\rho_t \psi)}{\partial t} = -\vec{\nabla}(\rho_t \psi \vec{u}) + S_\psi$$

$$\Downarrow$$

$$\frac{\partial \psi}{\partial t} = -\vec{u}.\vec{\nabla}\psi - \frac{\psi}{\rho_t}\left(\frac{\partial \rho_t}{\partial t} + \vec{u}.\vec{\nabla}\rho_t + \rho_t \vec{\nabla}(\vec{u})\right) + \frac{S_\psi}{\rho_t}$$

Error (in %) in the computation of "dry" mixing ratios of tracers (for example CO_2) from specific ratios

Tracer in an explicit cumulonimbus: before/after



Generalisation to a net sub-grid transport of total mass in deep convection parametrisations

HYMACS (Kuell, Gassmann and Bott, 2007), also Grell 3D in WRF

In the grey zone⇒ replace the parametrised compensating subsidence in the convective column by an "explicit" 3D subsidence computed by the dynamics.

Net mass advection inside the physics

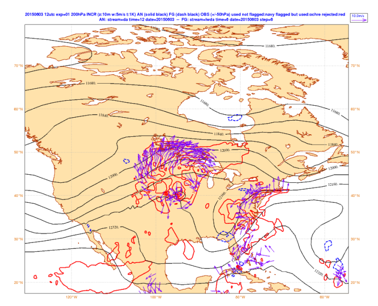
$$\frac{\partial(\rho\psi)}{\partial t})_{conv} = -\frac{\partial(M_u(\psi_u - \overline{\psi})}{\partial z} = -[\frac{\partial(M_u\psi_u)}{\partial z} + \frac{\partial(-M_u\overline{\psi})}{\partial z}]$$

$$\Downarrow$$

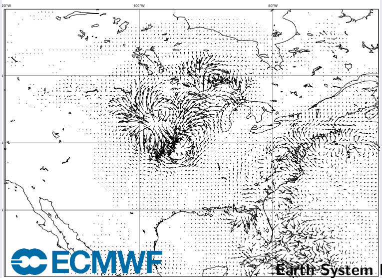
$$\frac{\partial(\rho\psi)}{\partial t})_{conv} = -\frac{\partial(M_u\psi_u)}{\partial z}$$

If the compensating subsidence is not parametrised by the convection scheme ⇒ the dynamics is expected to produce the mass adjustment as a consequence of the net mass transport by the physics;

MCs case : 3 June 2015, 00UTC, wind at 200hPa



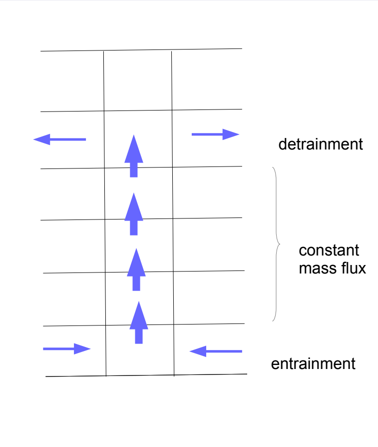
IFS increment computed by 4DVAR data assimilation.



200hPa Wind difference between simulations without and with deep convection scheme

How does the dynamics react to a parametrised net mass transport?

Academic simulation of a tracer transport by a single column updraft



- subgrid mixing of a passive tracer (not part of the total mass, not part of the buoyancy, unlike moisture)
- passive tracer with same initial profile as moisture
- no wind, temperature and moisture from Klemp and Wilhelmsom,78
- only the concentration of traceur + total mass in the new scheme are transported by the mass flux (temperature and moisture of parcels adjust instantaneously to the environment, not energetically correct, but only for illustration purpose)
- the vertical integral of the mass tendencies in the column is zero.

How does the dynamics react to a parametrised net mass transport?

- Hydrostatic model,
- Small planet, cubic grid, $dx = 5$ km, $dt = 1$ min

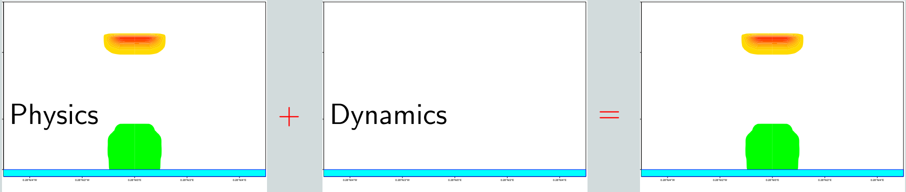
New mass flux scheme

$\Rightarrow$ change total mass with mass flux tendencies using same modifications written for total water tendencies (cf moist/dry mass conservation)

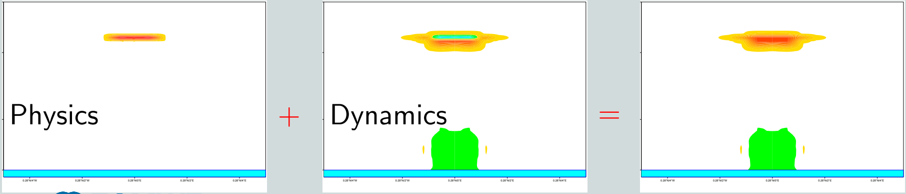
How does the dynamics react to a parametrised net mass transport?

6h. accumulated tendencies for the specific ratio of a passive tracer

Conventional mass flux scheme



New mass flux scheme



How does the dynamics react to a parametrised net mass transport?

- Non hydrostatic model,
- Small planet, cubic, $dx = 5$ km, $dt = 1$ min

NH-spectral IFS

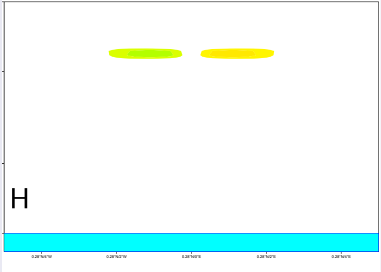
Hybrid mass vertical coordinate (π_s is the hydrostatic surface pressure)
Thermo equation for internal energy instead of enthalpy
Two more equations : $\hat{q} = \log(\frac{\hat{p}+\pi}{\pi})$ and d_4 related to the "vertical divergence".

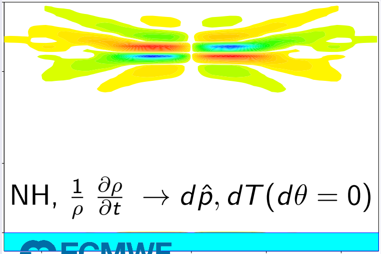
New mass flux scheme

$\Rightarrow$ mass flux tendencies projected on NH pressure departure
$d\hat{p} = d(p - \pi)$ and temperature dT such as $d\theta = 0$ (Kuell et al, 2007)

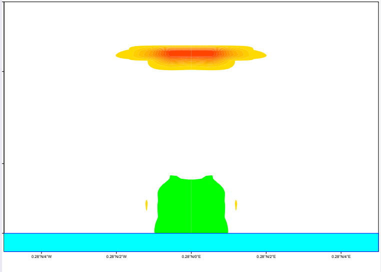
How does the dynamics react to a parametrised net mass transport?

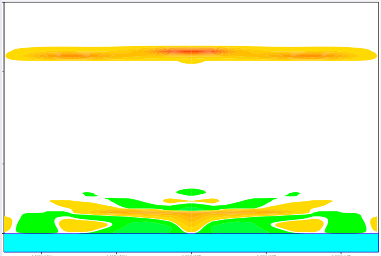
Wind after 1 k





Acc. tracer tend. after 6 hours





How does the dynamics react to a parametrised net mass transport?

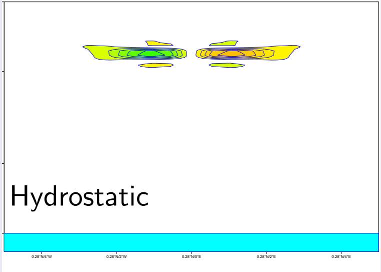
- Non-hydrostatic model
- Small planet, cubic, $dx = 5$ km, $dt = 1$ min

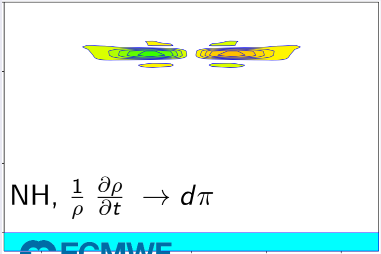
New mass flux scheme

$\Rightarrow$ change total mass with mass flux tendencies as in hydrostatic model (physics tendency for hydrostatic surface pressure, ω and $\dot{\eta}$)

How does the dynamics react to a parametrised net mass transport?

Wind after 1 k





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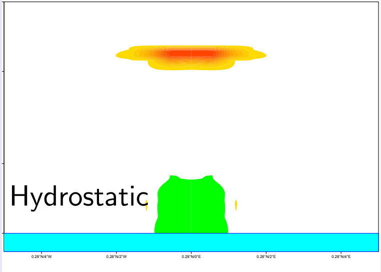
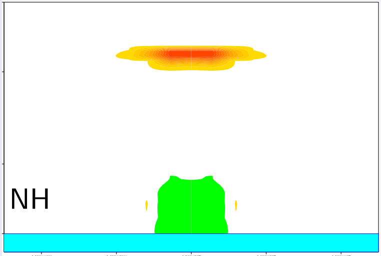




How does the dynamics react to a parametrised net mass transport?

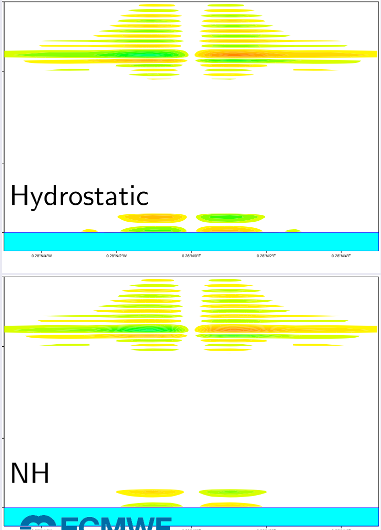
- Non hydrostatic model,
- Small planet, cubic, $dx = 500$ m, $dt = 5$ s

New mass flux scheme

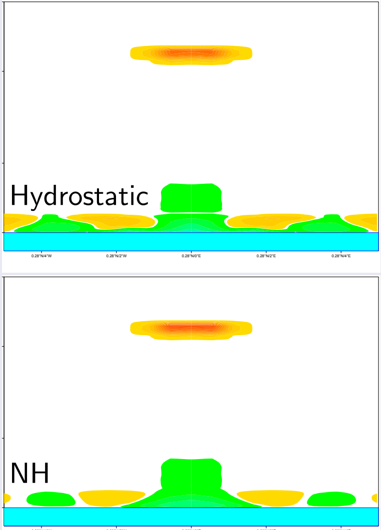
$\Rightarrow$ mass flux tendencies projected as in hydrostatic model (physics tendency for hydrostatic surface pressure, ω and $\dot{\eta}$)

How does the dynamics react to a parametrised net mass transport?

Wind after 1 k



Acc. tracer tend. after 3 hours



How does the dynamics react to a parametrised net mass transport?

- Hydrostatic model
- Small planet, cubic, $dx = 5$ km, $dt = 1$ min

New mass flux scheme

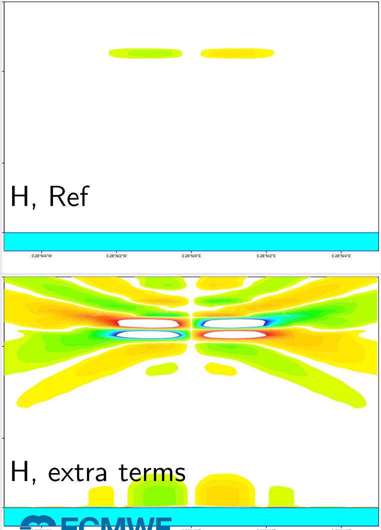
$\Rightarrow$ change total mass with mass flux tendencies as in hydrostatic model (physics tendency for hydrostatic surface pressure, ω and $\dot{\eta}$)

$$+$$

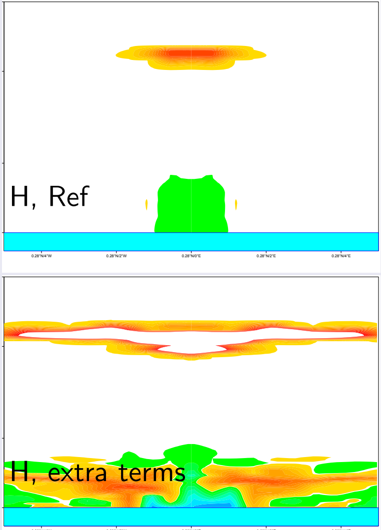
$$\frac{\partial \psi}{\partial t} = -\vec{u}.\vec{\nabla}\psi - \frac{\psi}{\rho_t}\left(\frac{\partial \rho_t}{\partial t} + \vec{u}.\vec{\nabla}\rho_t + \rho_t\vec{\nabla}(\vec{u})\right) + \frac{S_\psi}{\rho_t}$$

How does the dynamics react to a parametrised net mass transport?

Wind after 1 k



Acc. tracer tend. after 6 hours



Summary

- It is possible to conserve dry mass instead of total mass in the hydrostatic IFS. Neutral in term of scores, improve "dry" mixing ratios for tracers.
- It is also possible to parametrise a net sub-grid mass transport (e.g. mass flux scheme) and let the dynamics do the compensating subsidence. The same solution works for both the hydrostatic and NH IFS.
- But it is dangerous to play with mass!