An ensemble forecasting tutorial based on Lorenz-95 systems

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1. Introduction

a. The Lorenz-95 systems

This tutorial permits to run ensemble forecasting experiments using as system and forecast model variations of the system of ordinary differential equations introduced by Edward Lorenz in 1995. In all experiments, the unperturbed forecast model is given by

$$\frac{dx_i}{dt} = -x_{i-2}x_{i-1} + x_{i-1}x_{i+1} - x_i + F - g_U(x_i), \tag{1}$$

where $i = 1, 2 \ldots K$, and cyclic boundary conditions are used $x_0 = x_K,\ x_{-1} = x_{K-1},\ x_{K+1} = x_1$. Here, a 40-dimensional system $K = 40$ is considered. The magnitude of the forcing is set to $F \approx 8$ (see Table below). For this forcing the system is chaotic, i.e. it has positive Lyapunov exponents. Lorenz (1995) concluded that similar error growth characteristics to operational NWP systems are obtained if a time unit in the L95-system is associated with 5 days. All graphics use the non-dimensional time, i.e. the end of the forecast range of 4 corresponds to a 20-day forecast. Solutions of the system are obtained by numerical integration with a fourth-order Runge-Kutta scheme using a timestep of $\Delta t = 0.025$ (corresponds to 3 h). For the chosen forcing and $N = 40$, the system has 13 positive Lyapunov exponents, the largest corresponds to a doubling time of 2.1 days. The growth of small amplitude initial perturbations is illustrated in Fig. 1.

The dynamics is the same for each variable as eqn. (1) is invariant under the transformation $i \rightarrow i + 1$. Variables fluctuate about the mean in a non-periodic manner with a climatological standard deviation of $\sigma_{\text{clim}} \approx 3.6$. A perturbation of the initial condition will grow with time and its leading edge propagates “eastward” (to higher indices) at a speed of about 25 degrees/day — this corresponds to a shift of 14 indices in a (non-dimensional) time unit. See Lorenz (1995), Lorenz and Emanuel (1998) and Lorenz (2005) for a more detailed discussion of the system.

The “real atmosphere” is represented by the following system

$$\frac{\mathrm{d}x_k}{\mathrm{d}t} = -x_{k-1}\left(x_{k-2} - x_{k+1}\right) - x_k + F - \frac{hc}{b} \sum_{k=J(k-1)+1}^{Jk} y_j \tag{2}$$

$$\frac{\mathrm{d}y_j}{\mathrm{d}t} = -cby_{j+1}\left(y_{j+2} - y_{j-1}\right) - cy_j + \frac{c}{b}F_y + \frac{hc}{b}x_{1+\lfloor\frac{j-1}{J}\rfloor} \tag{3}$$

with $k = 1, \ldots, K$ and $j = 1, \ldots, JK$. The x-variables are referred to as slow variables and the y-variables are referred to as fast variables (see `doc/presentation.pdf` or Lorenz (1995); Wilks (2005)). The **perturbed forecast model** is given by

$$\frac{\mathrm{d}x_k}{\mathrm{d}t} = -x_{k-1}\left(x_{k-2} - x_{k+1}\right) - x_k + F - g_U(x_k) + \eta_k(t). \tag{4}$$

Here, $K = 40, J = 8$ and

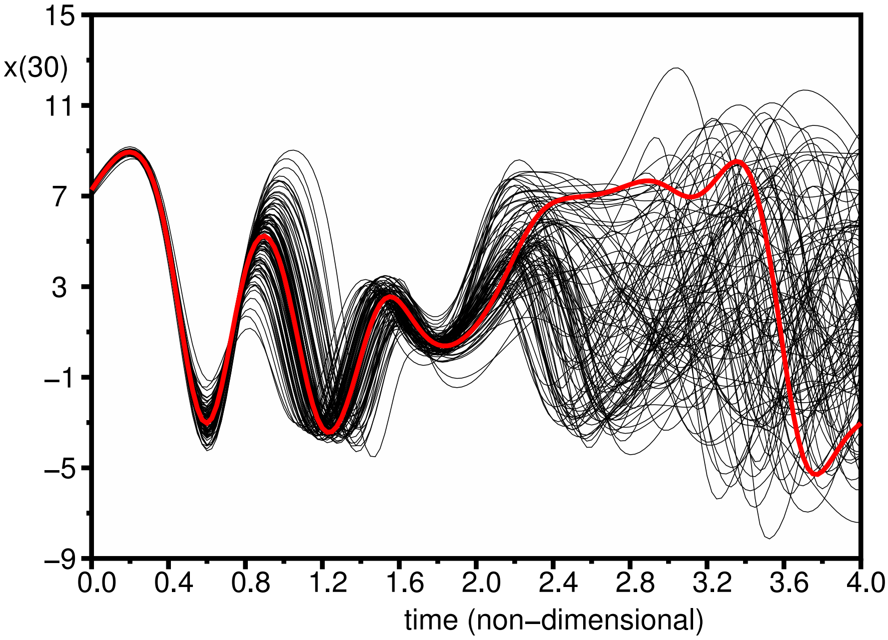

Figure 1: Evolution of the 40-variable Lorenz-95 system at site 30. Red solid line: unperturbed forecast; Black lines: ensemble of 100 forecasts which start from slightly perturbed initial conditions. The unperturbed initial conditions are an arbitralily selected state from a long integration of Eq. (1), i.e. a state on or close to the system attractor.

$b = 10$	**amplitude ratio** between slow variables and fast variables
$c = 10$	**time-scale ratio** between slow and fast variables
$h \in [0, 1]$	**coupling strength** between slow and fast variables, $h = 0$: uncoupled
$F = F_y \approx 8$	**forcing amplitude**
η_k	**stochastic forcing** with spatially uncorrelated and temporally correlated noise

The representation of model uncertainty through a stochastic forcing term follows the approach discussed by Wilks (2005).

b. Illustration of the dynamics of the forecast model

The chaotic dynamics of the Lorenz-95 system Eq. (1) can be explored by running (`l95demo.sci`). It displays an unperturbed solution and an ensemble in which the initial conditions have been perturbed (either locally at one site or globally at all sites).

2. Ensemble forecasting experiments

The programme `run_eps.sci` computes the ensemble forecasts for initial dates that are 0.4 time units apart (the equivalent of 2 days using Lorenz's scaling). It is possible to look at the

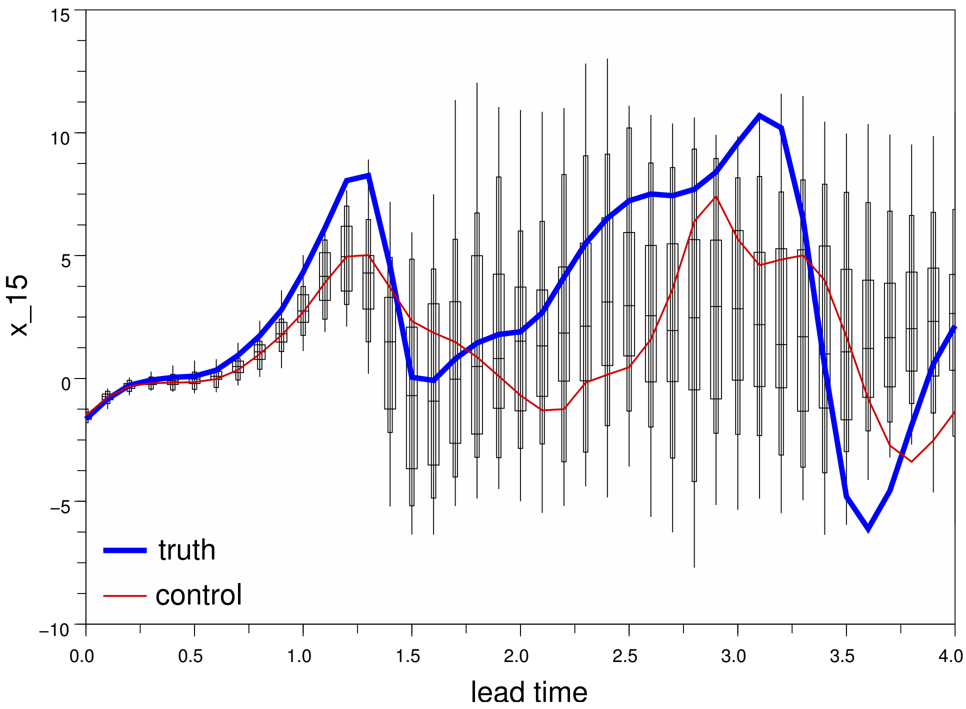

Figure 2: EPS-gram for site 15. This box-whisker diagram shows the min/max of the EPS distribution, and the 0.1, 0.25, 0.5, 0.75, 0.9 quantiles. The example is for an imperfect model scenario and a 50 member ensemble. A lead time of 1 corresponds to 5 days.

EPS-grams for each ensemble forecast (Fig. 2). After completing many cases (up to 180) some summary measures are displayed which quantify the statistical consistency and accuracy of the probabilistic forecasts.

3. Exercises

Specific exercises that guide you through various topics are given in `doc/presentation.pdf`.

4. Outlook

The tutorial is a first step towards an interactive tool to understand probabilistic forecasting. It would be of interest to link the ensemble forecasting system with an actual data assimilation system. Such experiments are discussed by Leutbecher et al. (2007). All components for this exercise are available; the interested reader is referred to the tutorial in the ECMWF data assimilation training course.

5. Feedback

Your comments on the tutorial are appreciated. I am interested to hear from you should you find any errors in this software.

References

Leutbecher, M., R. Buizza, and L. Isaksen, 2007: Ensemble forecasting and flow-dependent estimates of initial uncertainty. In *Flow-dependent aspects of data assimilation*, Workshop Proceedings, ECMWF, Shinfield Park, Reading, UK, 185–201. Available at www.ecmwf.int/publications/library/ecpublications/_pdf/workshop/2007/Data_assimilation.

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Wilks, D. S., 2005: Effects of stochastic parametrizations in the Lorenz '96 system. *Quart. J. Roy. Meteor. Soc.*, **131**, 389–407.

A How to install the tutorial

You are welcome to use this tutorial for educational purposes as long as you acknowledge its origin. The installation of the tutorial should be straightforward.

1. Install scilab (version 4.1 or higher is recommended). Scilab is free software available from `www.scilab.org`. Scilab is quite similar to the commercial software package matlab.
2. Obtain a copy of the tutorial software (distributed as gzipped tar-file `PR-TC.tar.gz`). You can e-mail me (M. Leutbecher "at" ecmwf.int) to receive a copy of the programmes in case they are not included in the training course CD-ROM.
3. Unpack the archive `PR-TC.tar.gz` in a suitable directory
4. Create the data used by the tutorial.
   - On UNIX architectures:

     ```
     cd PR-TC/sci
     make
     ```

   - On Windows: start scilab and execute the scilab programmes called in `PR-TC/sci/makefile`
5. start the tutorial:
   - On UNIX, execute `PR-TC/bin/tutorial`;
   - On Windows, start scilab, change directory to `PR-TC/sci`, execute `init.sci`