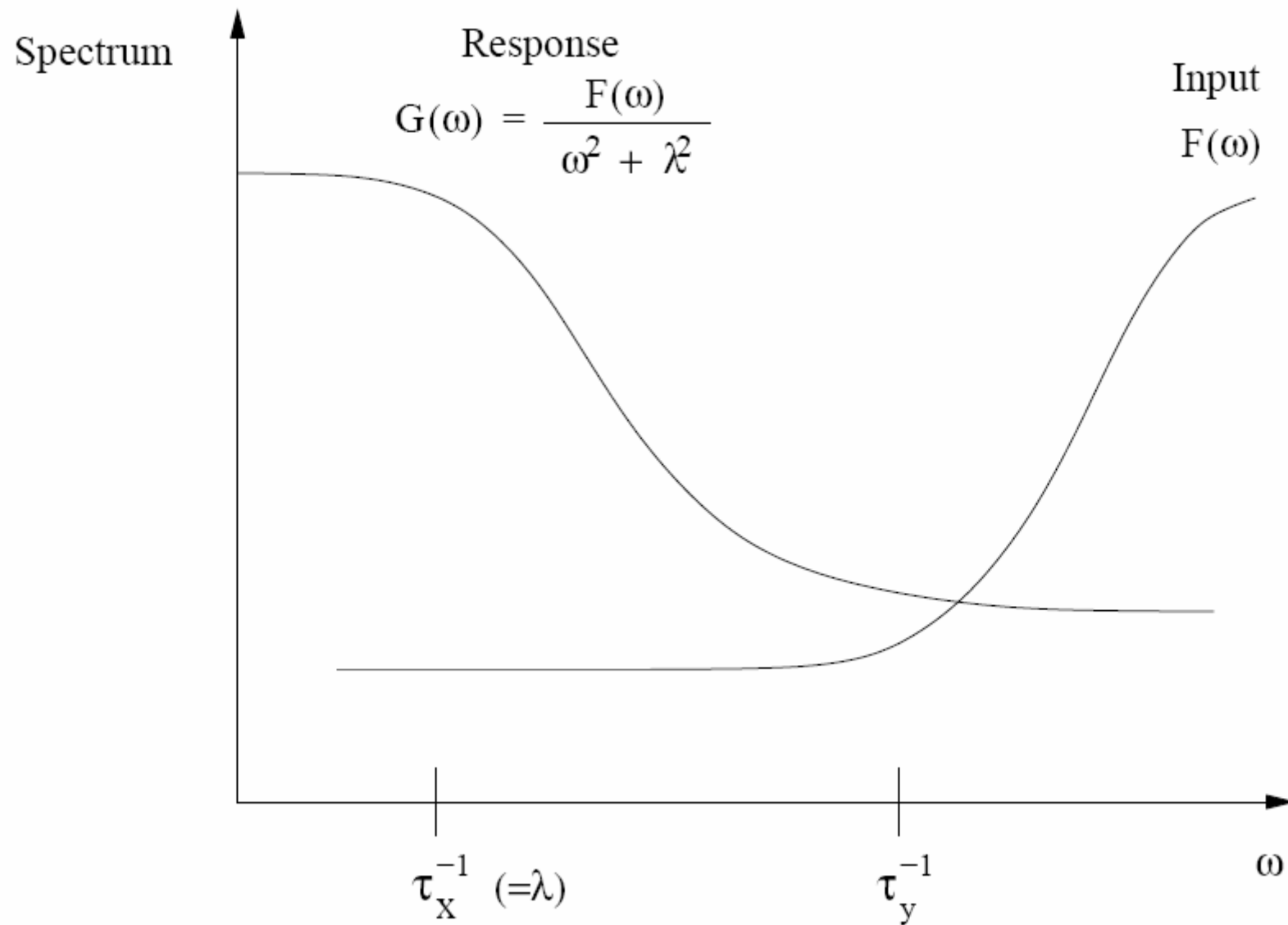


Sensitivity of a Double-Gyre Ocean Model to “Details” of Stochastic Forcing

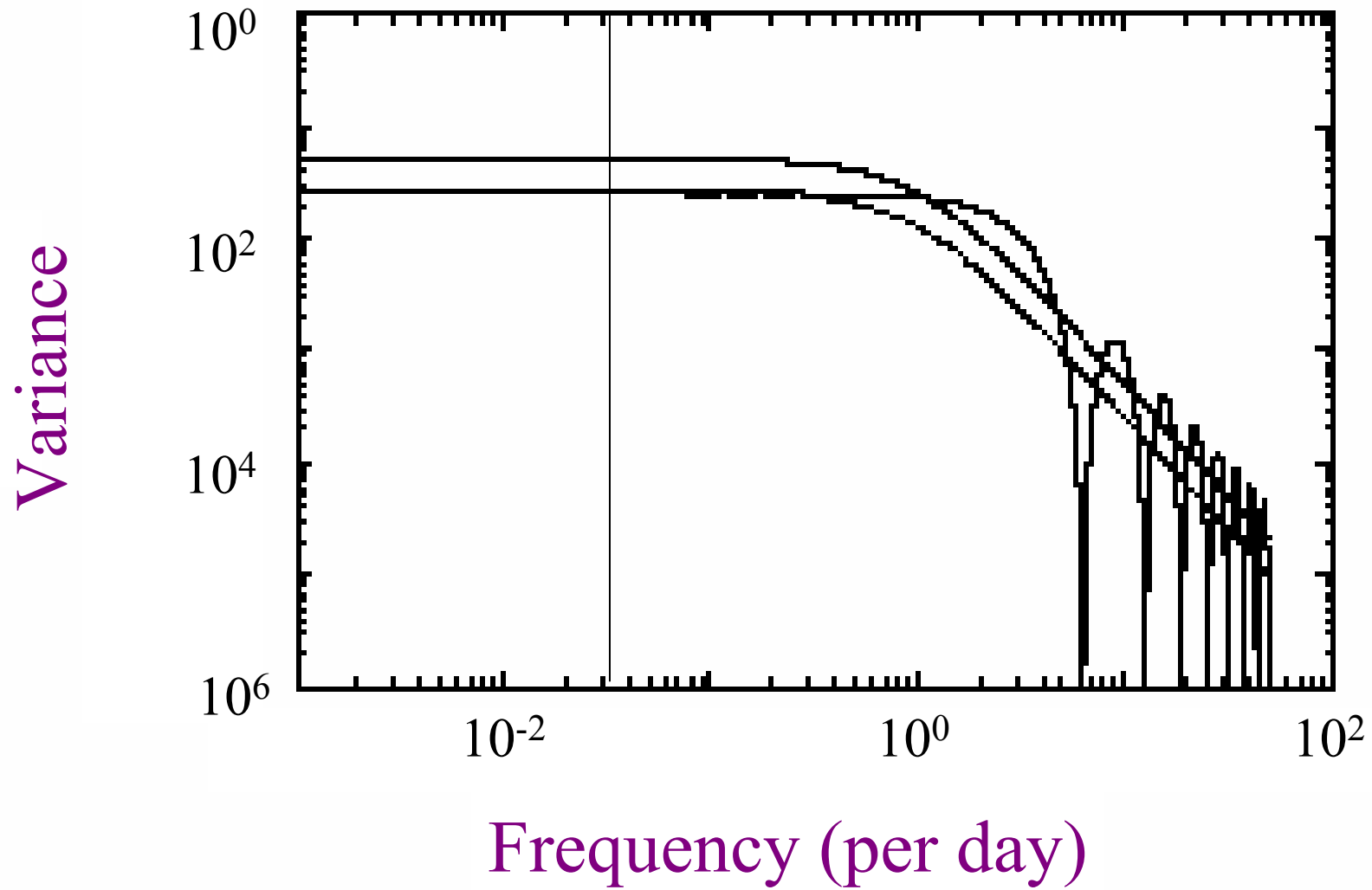
Cécile Penland and Philip Sura

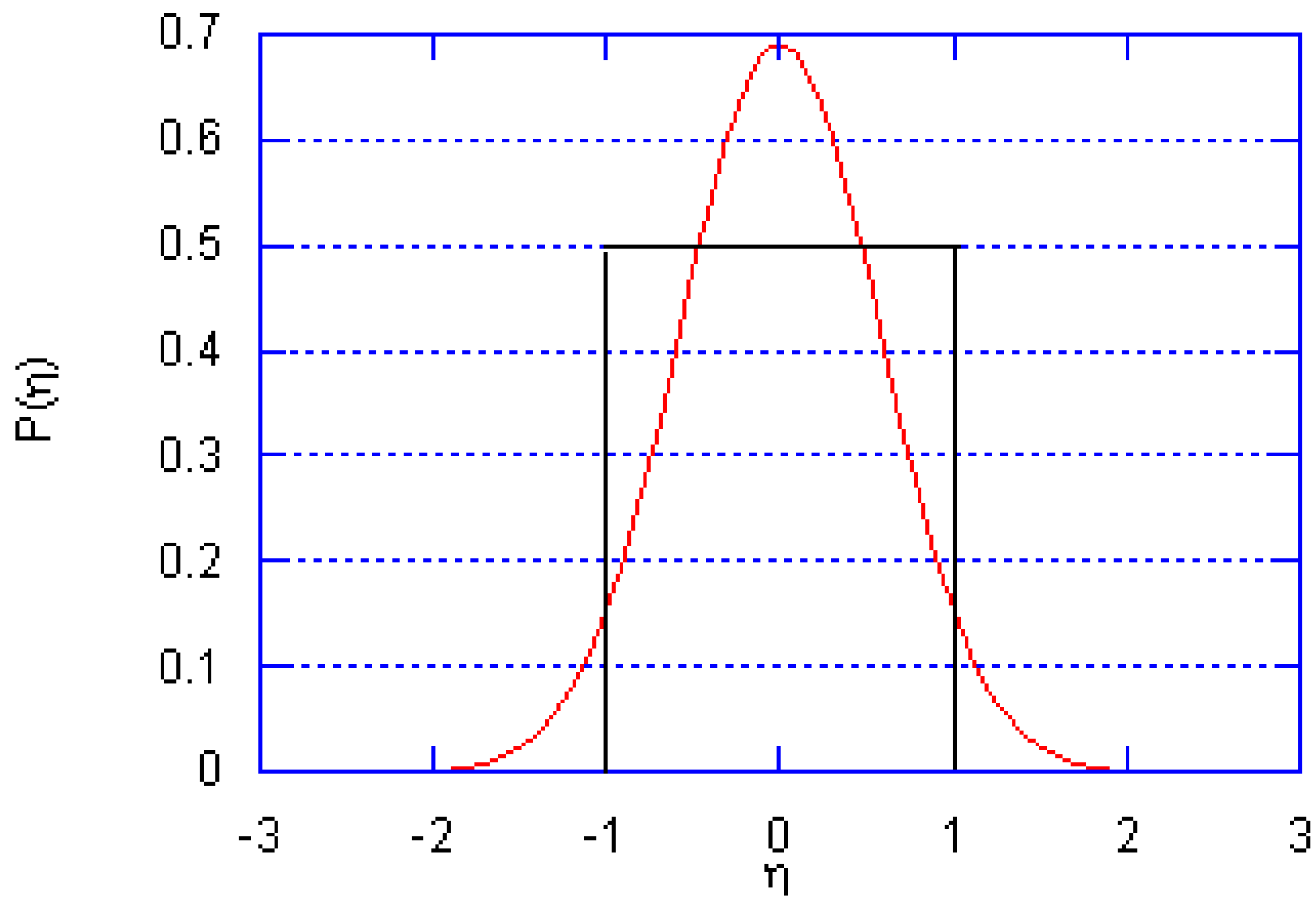
NOAA-CIRES/Climate Diagnostics Center

Hasselmann (1976)



Spectra of different forcing functions





There is some reason for thinking the Gaussian might be more appropriate:

- Our model: $d\mathbf{x}/dt = \varepsilon^2 \mathbf{G}(\mathbf{x},t) + \varepsilon \mathbf{F}(\mathbf{x},t)$

$\varepsilon^2 \mathbf{G}(\mathbf{x},t)$ is slow

$\varepsilon \mathbf{F}(\mathbf{x},t)$ is fast

The Central Limit Theorem.

Choose a scaling $s = \varepsilon^2 t$:

$$\frac{d\mathbf{x}}{ds} = \mathbf{G}(\mathbf{x}, s/\varepsilon^2) + \frac{1}{\varepsilon} \mathbf{F}(\mathbf{x}, s/\varepsilon^2) \quad (*)$$

For simplicity, say

$$F_i(\mathbf{x}, s/\varepsilon^2) = \sum_k F_i^k(\mathbf{x}, s) \eta^k(s/\varepsilon^2) \quad \text{and}$$

$$C_{km} = \int_{-\infty}^{\infty} \langle \eta_k(t) \eta_m(t'+t) \rangle dt' \equiv (\phi \phi^T)_{km}$$

$$\lim_{\substack{t \rightarrow \infty \\ \varepsilon \rightarrow 0}} (*) \rightarrow d\mathbf{x} = \mathbf{G}(\mathbf{x}, s) ds + \sum_{k, \alpha} \mathbf{F}^k(\mathbf{x}, s) \phi_{k\alpha} \bullet dW_\alpha$$

Reduced-gravity Ocean Model:

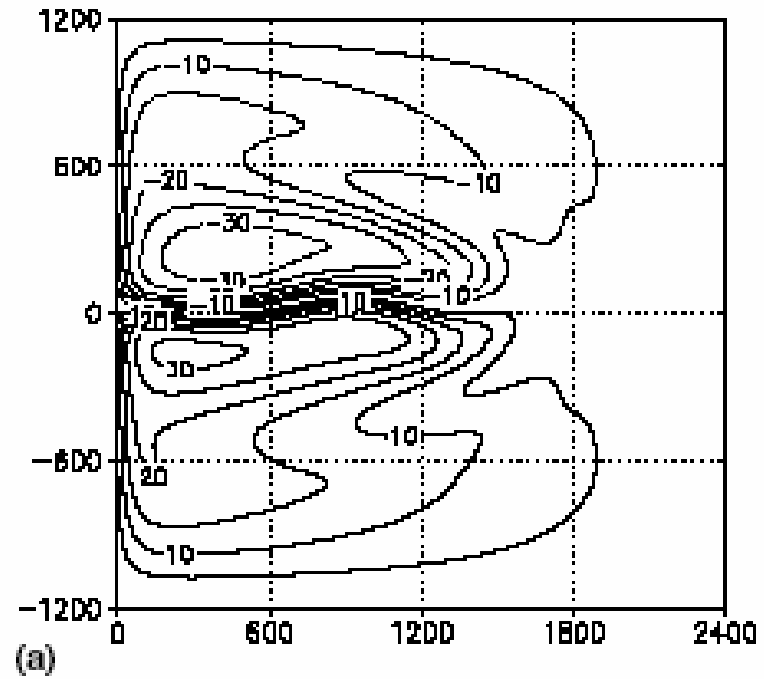
$$\frac{\partial U}{\partial t} + \frac{\partial}{\partial x} \left(\frac{U^2}{H} \right) + \frac{\partial}{\partial y} \left(\frac{UV}{H} \right) - fV = \frac{-g'}{2} \frac{\partial H^2}{\partial x} + \frac{\tau^x}{\rho_1} - rU + A\Delta U, \quad (1)$$

$$\frac{\partial V}{\partial t} + \frac{\partial}{\partial x} \left(\frac{UV}{H} \right) + \frac{\partial}{\partial y} \left(\frac{V^2}{H} \right) + fU = \frac{-g'}{2} \frac{\partial H^2}{\partial y} + \frac{\tau^y}{\rho_1} - rV + A\Delta V, \quad (2)$$

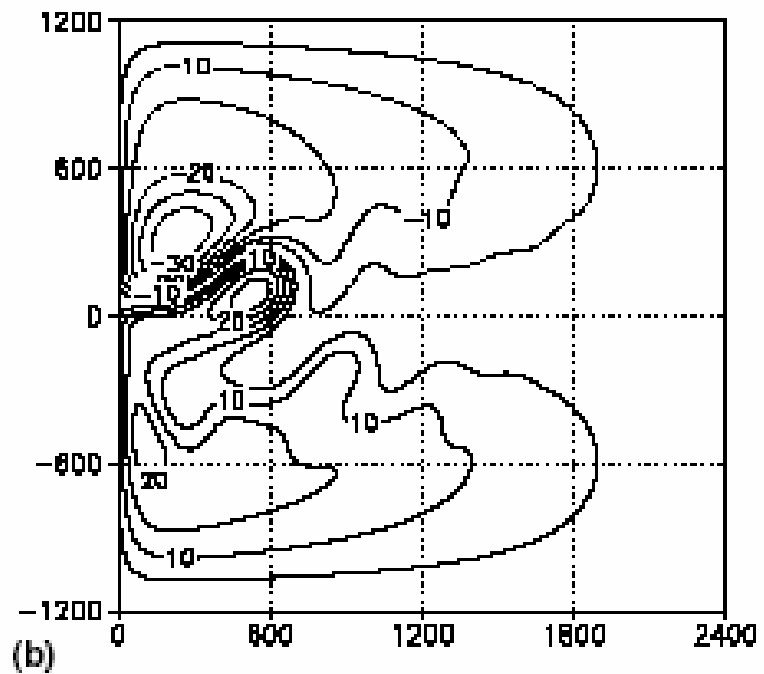
$$\frac{\partial H}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0, \quad (3)$$

Equations describe top layer of thickness H overlying a deep, motionless layer. Use beta-plane approximation for f . (Sura and Penland 2002).

Low KE state



High KE state



$$\tau = \tau_{\text{Mean}} + \tau_{\text{Stochastic}}$$

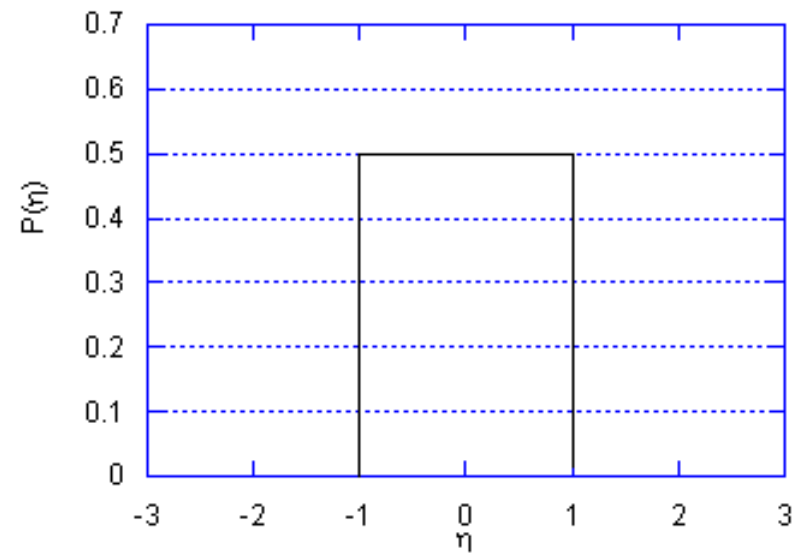
τ_{Mean} is proportional to $\cos(2\pi y/L)$

$$\tau_{\text{Stochastic}} = \rho_{\text{air}} |\mathbf{u}'| \mathbf{u}'$$

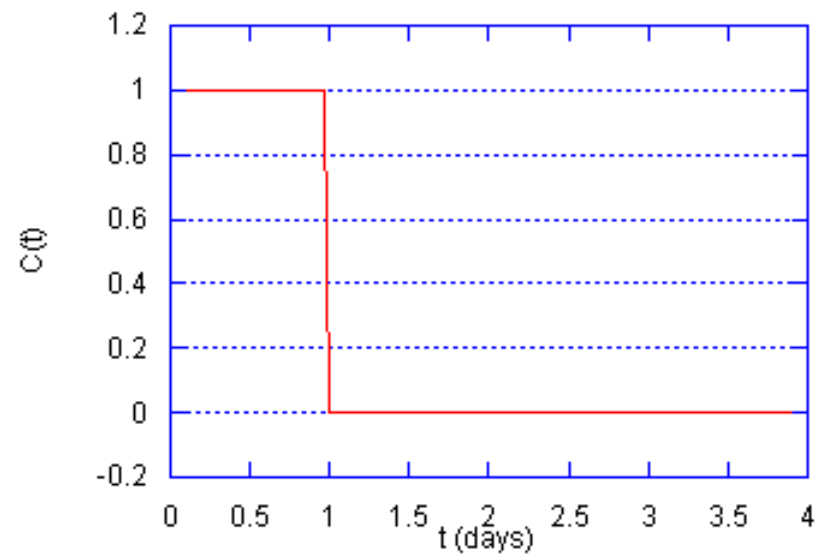
$u_i' = \eta_i'(t)f(x,y)$, $f(x,y)$ is Gaussian

Four models for stochastic $\eta_i'(t)$

Model 1:

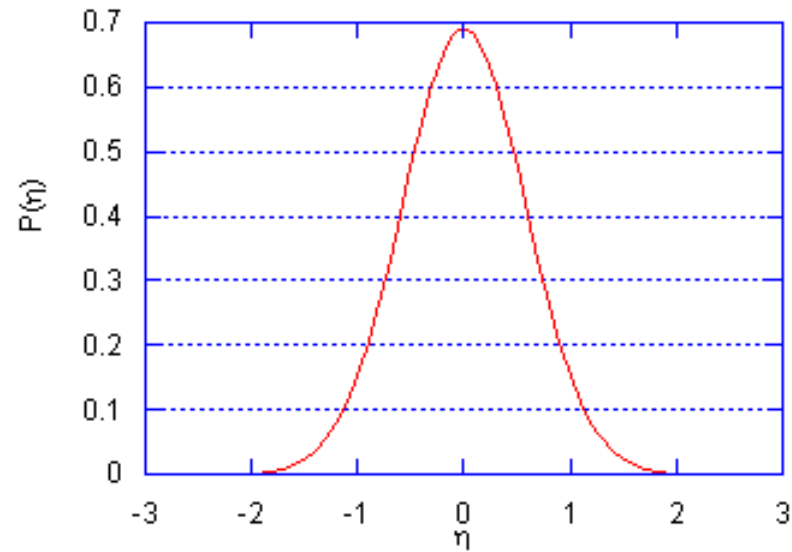


Distribution
is flat.

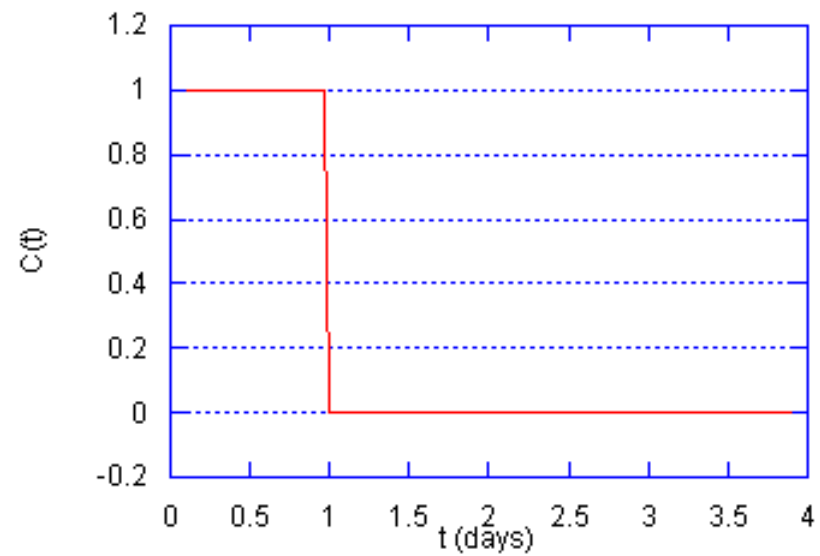


Correlation
is flat.

Model 2:

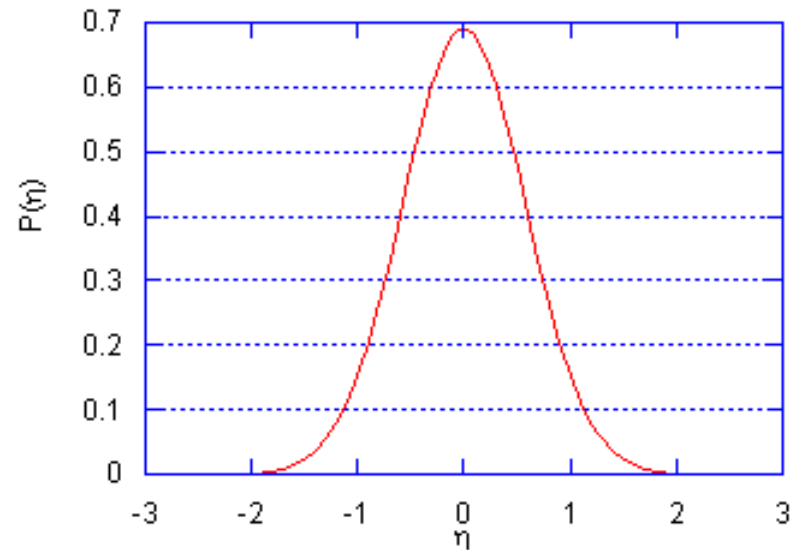


Distribution
is Gaussian.

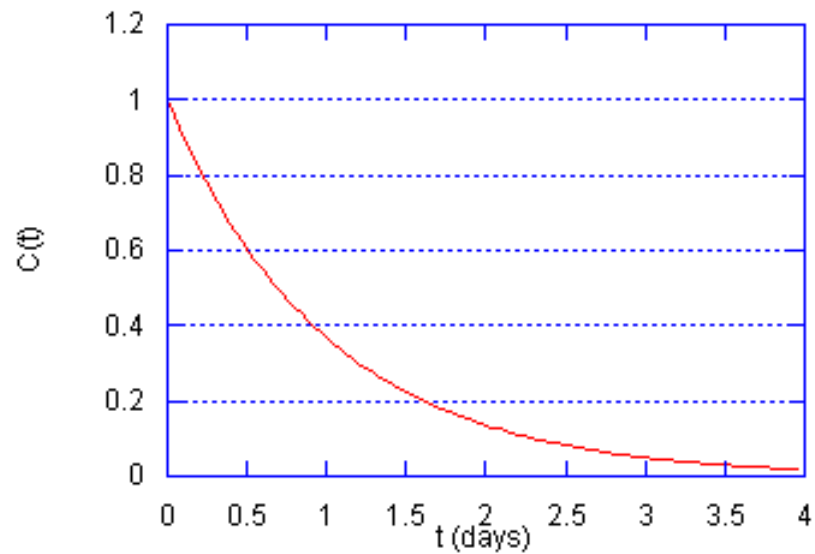


Correlation
is flat.

Model 3:
(OU process)



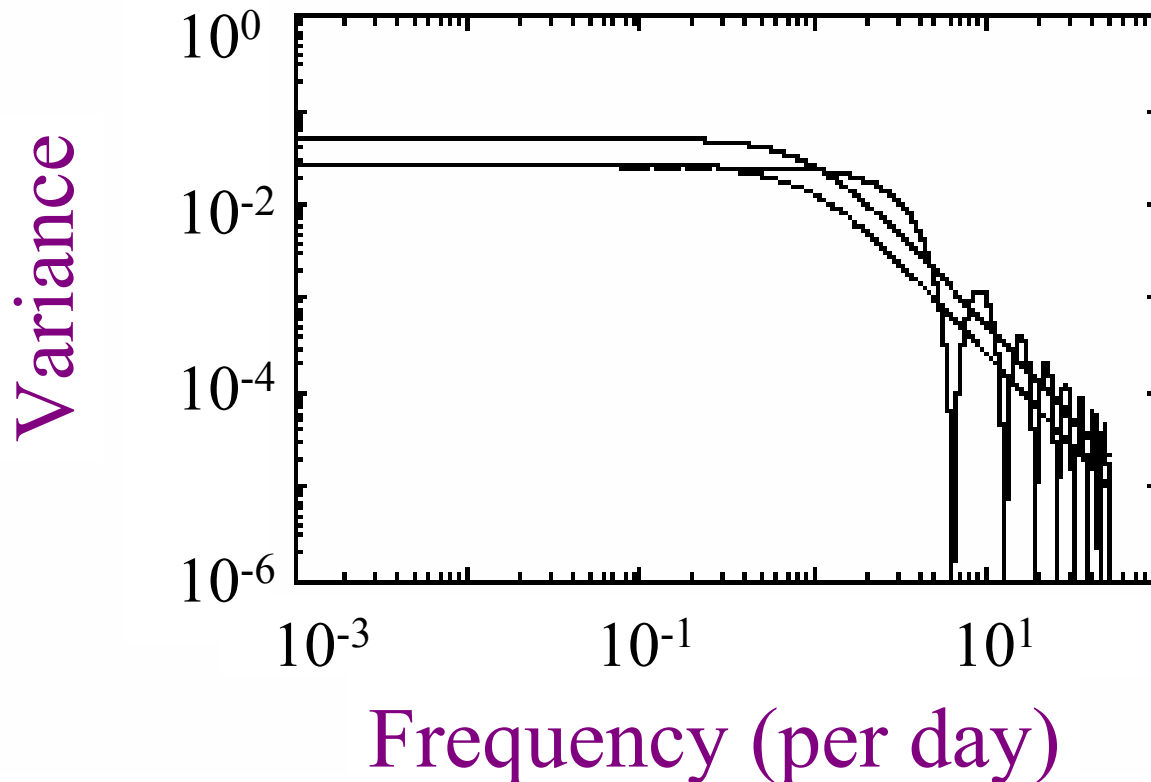
Distribution
is Gaussian.



Decay time
is one day.

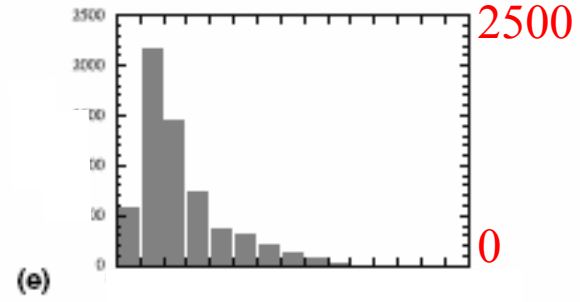
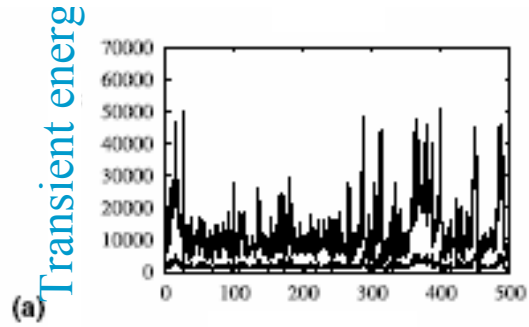
Model 4: Same as model three but with variance chosen to match low-frequency spectrum of Models 1 and 2.

Spectra of different forcing functions

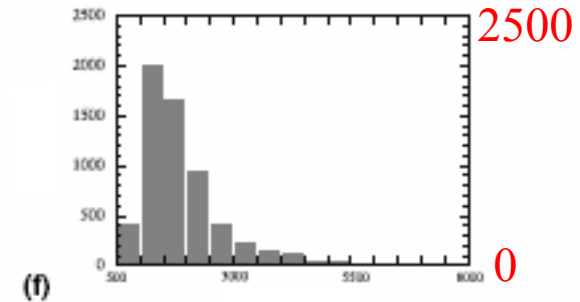
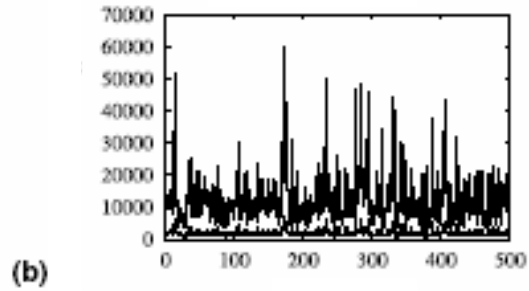


Monthly average model output!

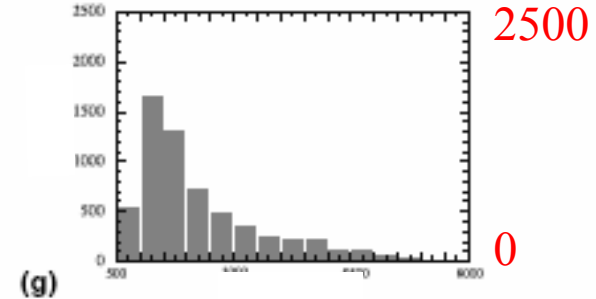
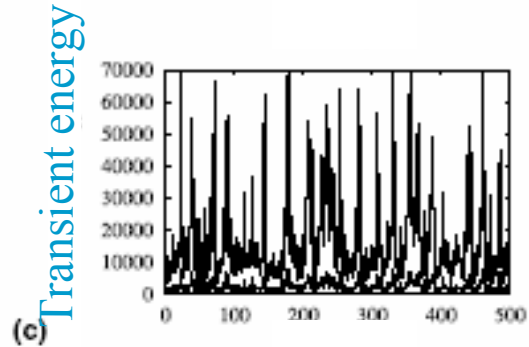
Model 1
(Control)



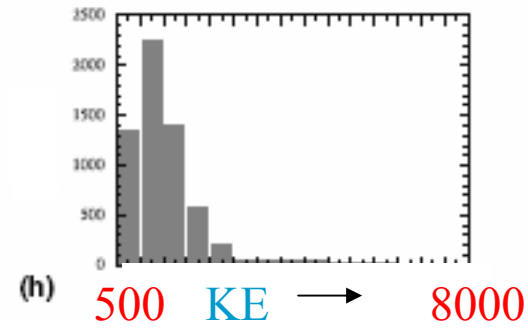
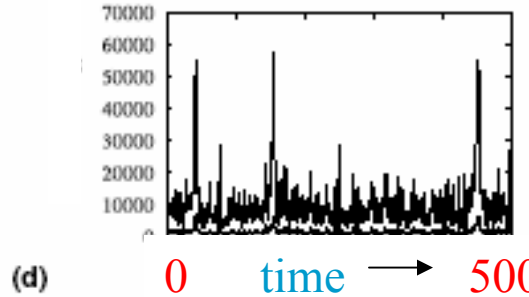
Model 2



Model 3

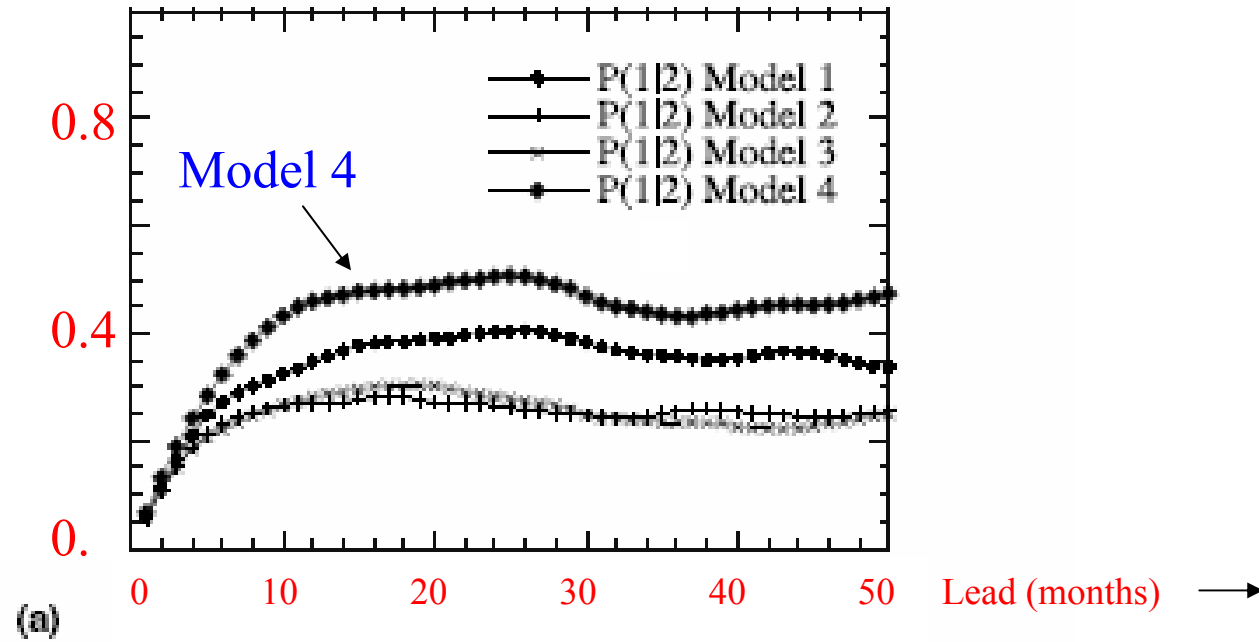


Model 4

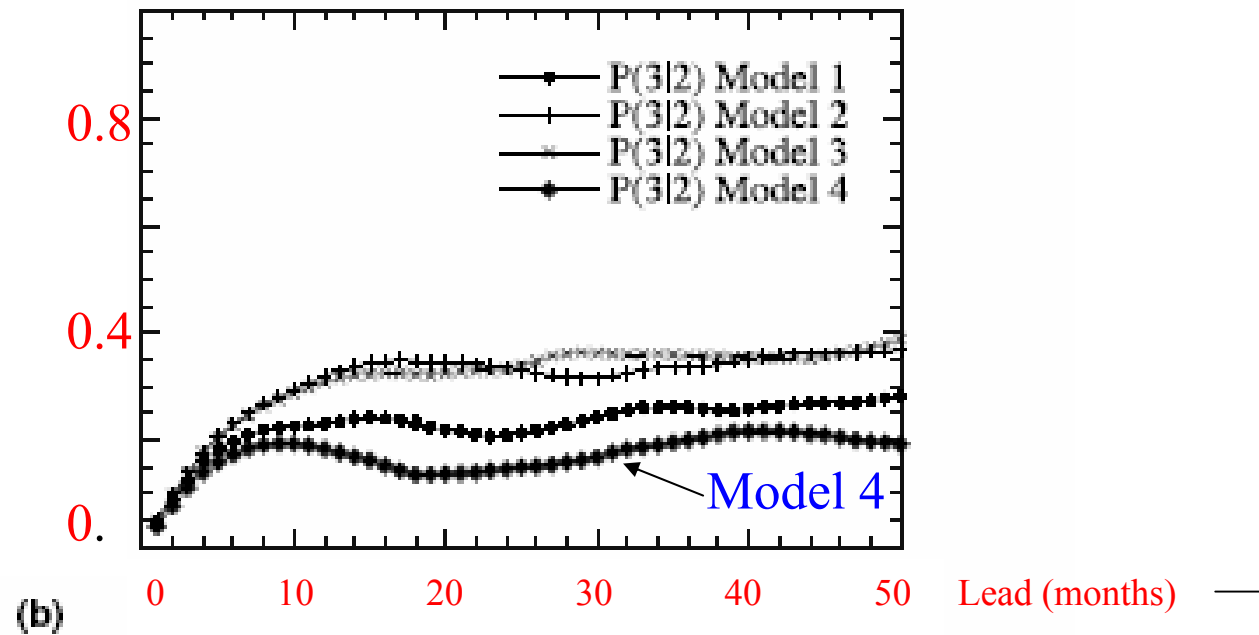


Histograms of Transient KE

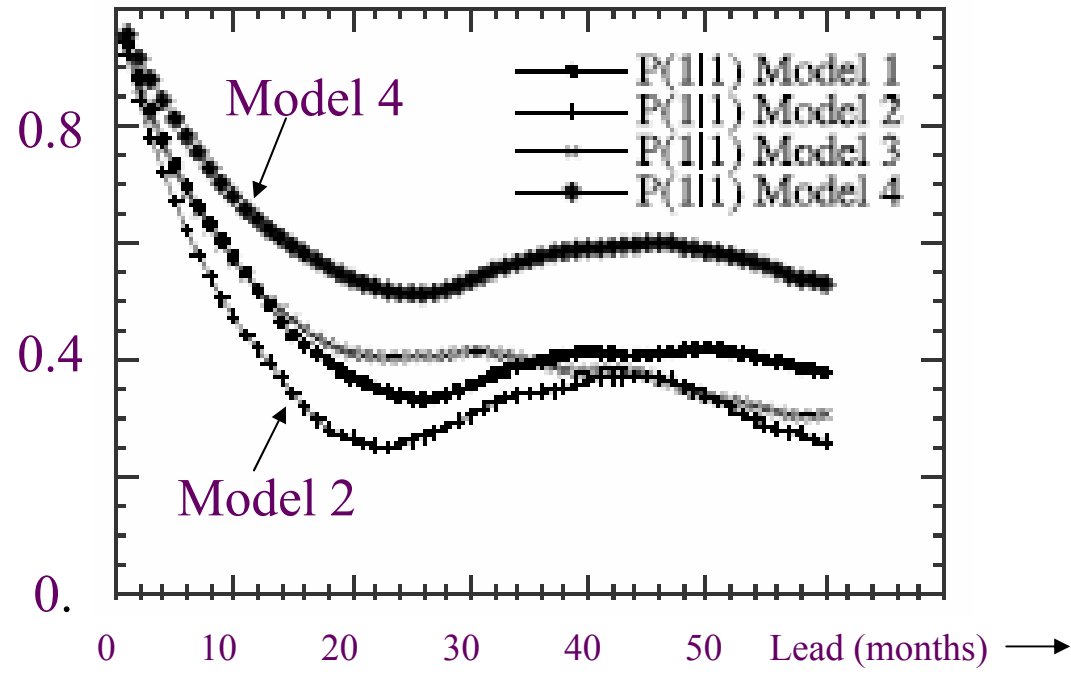
Probability of transition from middle to lower tercile.



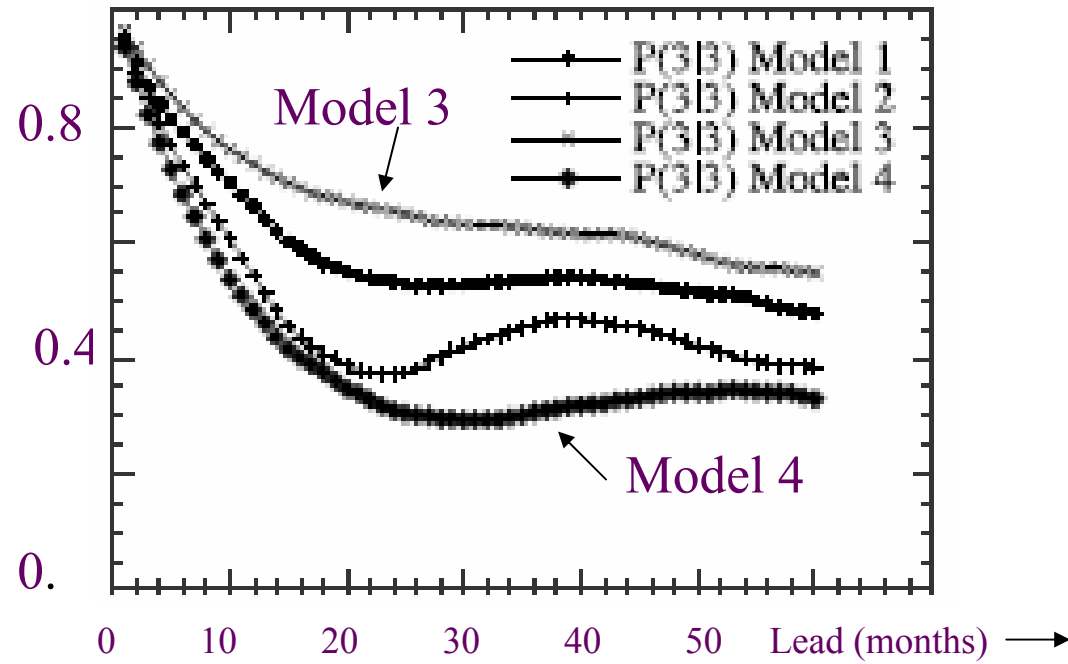
Probability of transition from middle to higher tercile.



Recurrence Probability



Recurrence Probability



Conclusions

- Details of the forcing at short timescales affect long timescales
- Pay attention to the distribution as well as the spectrum of the noise.
- Gaussian tails allow this model to sample more variability than the flat distribution.