Coarse grained stochastic models for tropical convection and climate

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OUTLINE

- Introduction, unresolved features (CAPE and CIN)
- Stochastic spin/flip model and coarse-graining
- Deterministic model and coupling
- Results: stochastic Walker cell
- Summary

Relevant papers

- Atmospheric science:

  Khouider, Majda, & Katsoulakis (2003), PNAS: *Coarse grained stochastic models for tropical convection and climate.*

  Majda & Khouider (2002), PNAS: *Stochastic and mesoscopic models for tropical convections.*

- Coarse Graining (Material Science):

  Katsoulakis, Majda, & Vlachos (2003), JCP:

  Katsoulakis, Majda, & Vlachos (2003), PNAS

- Multiscale Coupling and Phase Transition

  Katsoulakis, Majda, & Sopasakis (2004): Deterministic closures

  Katsoulakis, Majda, & Sopasakis (2005): Stochastic closures

  Katsoulakis, M. A., A. J. Majda, and A. Sopasakis, (2005b), *Intermittency, metastability, ...*

Introduction

- Moist convection: Transport of latent heat.
- Source of energy for local and large scale circulation.
- Generates and maintains tropical waves and storms.
- Organized tropical convection ranges from mesoscale individual clouds (1-10 km) to large scale superclusters (1000-10,000 km).
- Poorly represented by GCM's despite Today's supercomputers.
- Major contemporary problem: How large-scale circulation supplies energy and maintains deep convection?
- Convective Inhibition (CIN): Energy Barrier for spontaneous convection

Motivation

- Can Stochastic parametrizations alter tropical Climatology?
- Can they increase the wave fluctuations?
- Lin & Neelin: suggest plausible influence of stochastic convective parametrizations on the variability in GCM's.

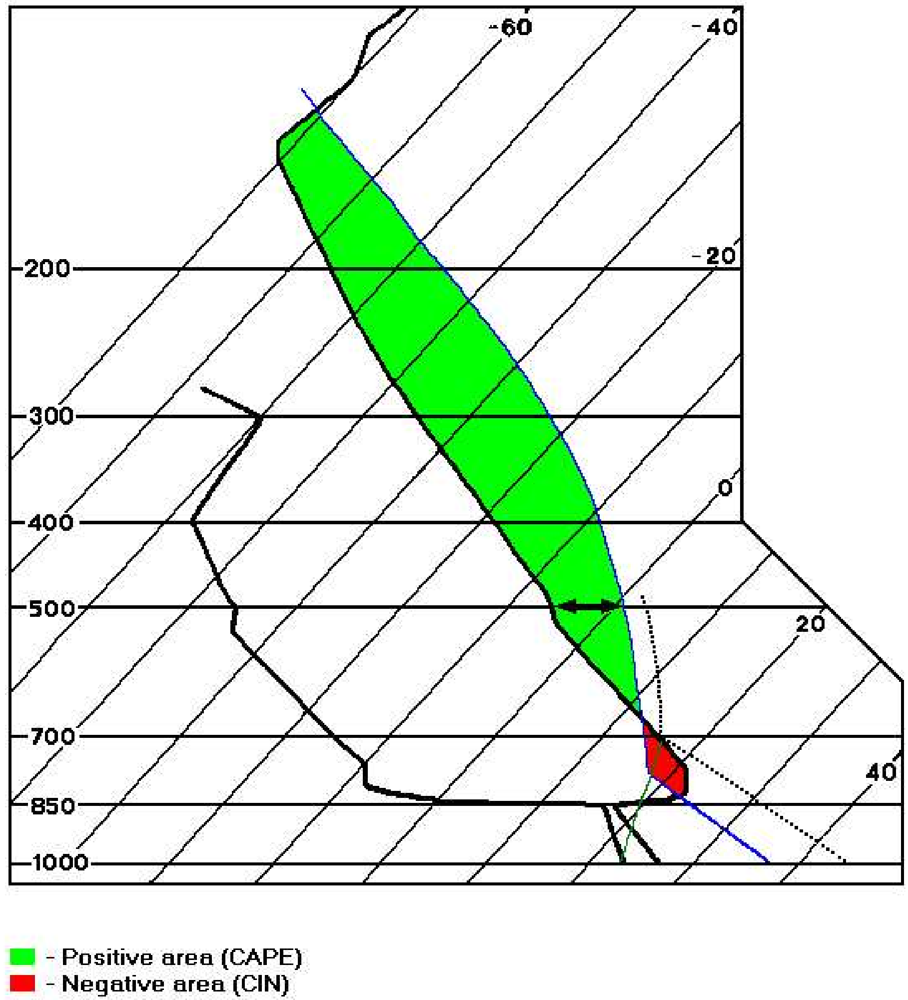
Static stability of Lifted parcel

Convective Inhibition (CIN): Energy Barrier for spontaneous convection.

Sounding (black)

Lifted parcel (blue line) cools by expansion,

at LCL: warms by latent heat release of condensation



Source: Internet.

- (Thermal) Buoyancy of a lifted parcel:

$$B = g\frac{\theta_{e,p} - \theta_{e,a}}{\theta_{e,a}}$$

θ_e = temperature + moisture content × latent heat

- Potential energy of lifted parcel

$$\begin{aligned} E_p &= \int_0^{LNB} g\frac{\theta_{e,p} - \theta_{e,a}}{\theta_{e,a}} dz \\ &= \int_0^{LFC} \text{---}\,dz + \int_{LFC}^{LNB} \text{---}\,dz \\ &= -\text{CIN} + \text{CAPE} \end{aligned}$$

- When (how) parcel has (could have) enough energy to overcome CIN and reach LFC?

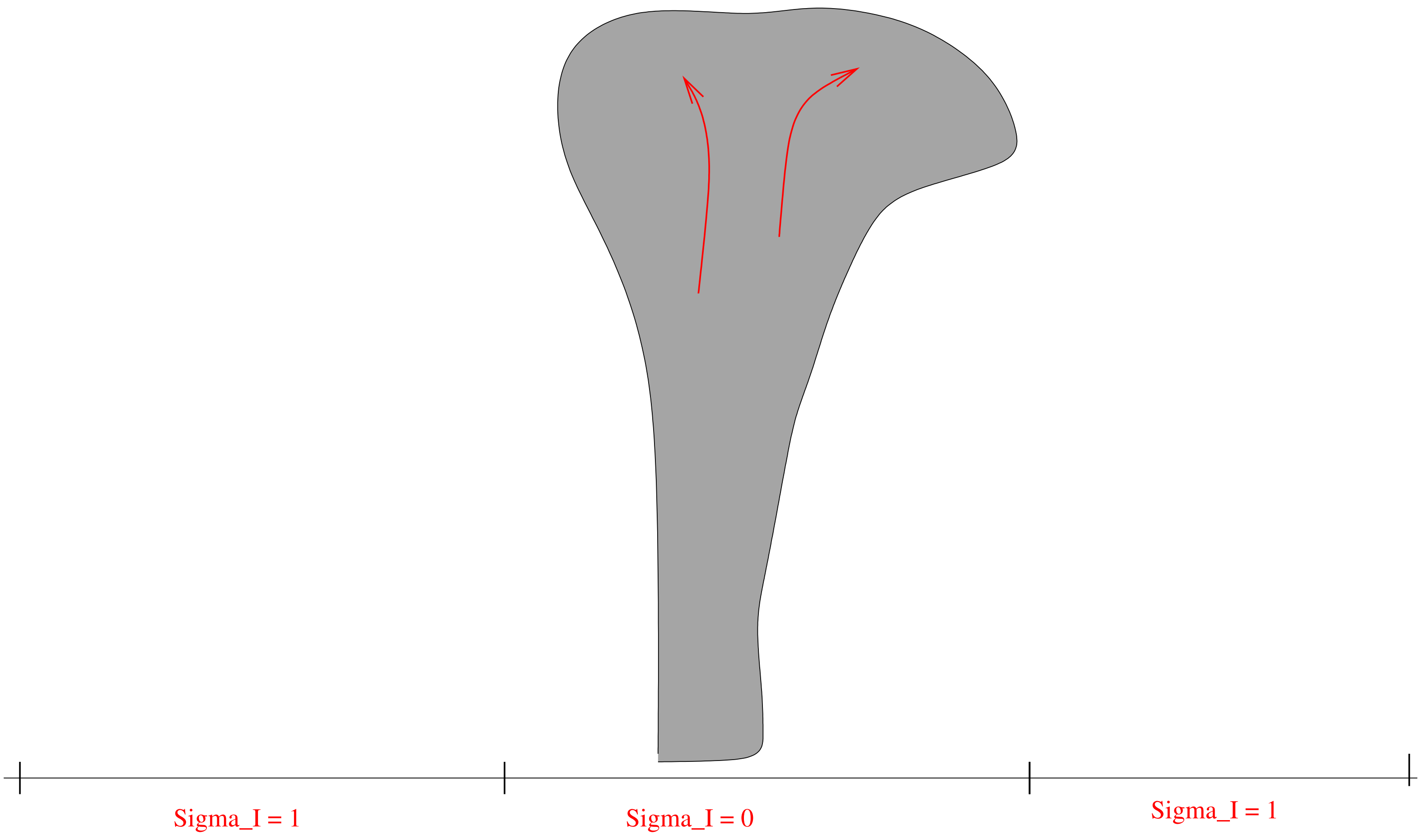
Microscopic stochastic Model for CIN

- CIN: Energy Barrier for spontaneous convection
- Observationally, factors for CIN complex:
  gust fronts, gravity waves, turbulent fluctuations in boundary layer equivalent potential temperature, etc.
- Our point of view:
  Too complex to model in detail; instead, borrow ideas from statistical physics and material science of representing these effects by an order parameter, σ_I

- Order parameter, σ_I, sites (1-10 km apart)

  $\sigma_I = 1$ if deep convection is inhibited: a CIN site

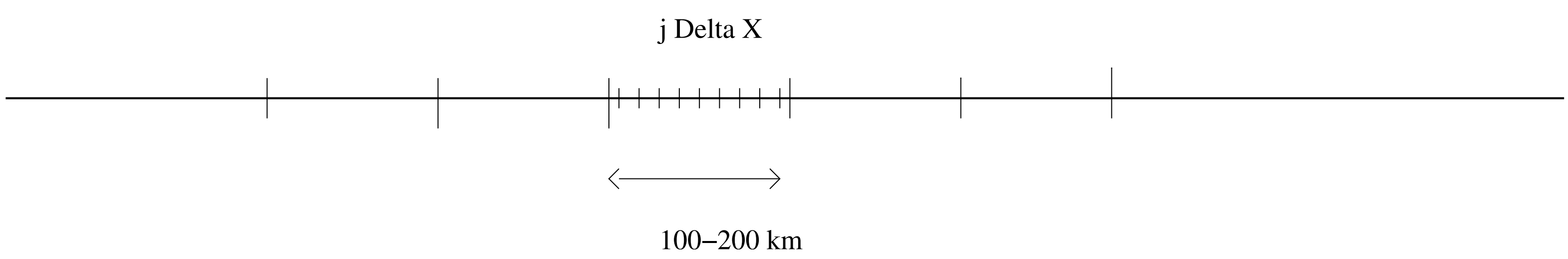
  $\sigma_I = 0$ if there is potential for deep convection: a PAC site



- Coarse Mesh: Average CIN

$j\Delta x, \Delta x = O(100, 200 \text{ km})$ (mesoscopic scale)

$$\bar{\sigma}_I(j\Delta x, t) = \frac{1}{\Delta x}\int_{(j-1/2)\Delta x}^{(j+1/2)\Delta x} \sigma_I(x,t)\,dx$$



Intuitive Stochastic Rules for Interaction of Order Parameter, σ_I

A) If CIN site is surrounded mostly by CIN sites, should remain so with high probability

B) If PAC site is surrounded by CIN sites, should have high probability to switch to CIN site

C) The external large scale mesoscopic mesh values, $\vec{u}_j$, should supply external potential, $h(\vec{u}_j)$, which modifies dynamics in A) and B) according to whether external conditions favor CIN or PAC

Stochastic Model

- View boundary layer as heat bath with External Potential:
  Ising model (magnetization and phase transition)
  Materials science: Souganidis, Katsoulakis, etc.

- Microscopic energy for CIN:

$$H_h(\sigma_I) = \sum_{x \neq y} J\left(\frac{|x-y|}{L}\right)\sigma_I(x)\sigma_I(y) + h \sum_x \sigma_I(x)$$

  – J: microscopic interaction potential: (Currie-Weiss)

$$J(r) = \begin{cases} U_0, & r < r_0 \\ 0, & r > r_0 \end{cases}$$

  – h: external potential. $H_h \nearrow h$

- Invariant Gibbs measure: $G = (Z_\Lambda)^{-1} \exp[\beta H_h(\sigma)] d\sigma$
  Z_Λ : partition function.

- Spin flip rule: $\sigma_I^x(y) = \begin{cases} 1 - \sigma_I(x), & y = x \\ \sigma_I(y), & y \neq x \end{cases}$
- Arrhenius dynamics:

  Rate: $c(x, \sigma_I) = \begin{cases} \tau^{-1} \exp[-\beta V(x)], & x = 1 \\ \tau^{-1}, & x = 0 \end{cases}$

  $V(x) \equiv \Delta H = \sum_{z \neq x} J(x - z)\sigma_I + h(x)$

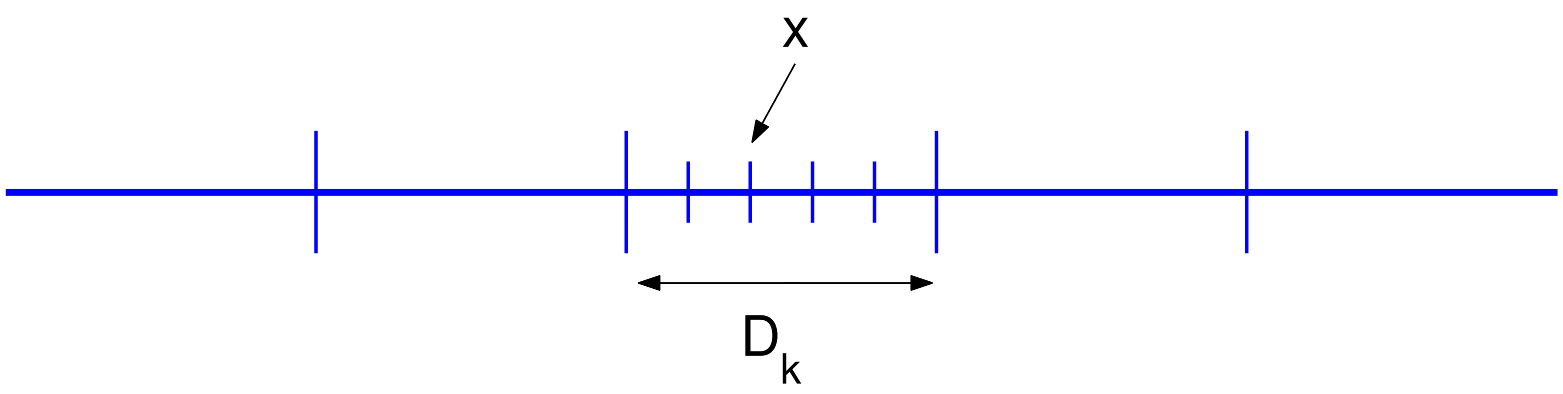
  τ_I : CIN characteristic time O(days)
- With $U_0 > 0$: if a CIN site is mostly surrounded by CIN sites, then it needs to overcome a larger energy barrier.
- External field also builds/destroys energy for CIN: $H_h \nearrow h$

Coarse-Graining

- Coarse-grained stochastic process:

$$\eta_t(k) = \sum_{x \in D_k} \sigma_{I,t}(x); \qquad \eta(k) \in \{0, 1, \cdots, q\}$$

average CIN on coarse-mesh: $\bar{\sigma}_I[D_k] = \frac{1}{q}\eta(k)$



Features of coarse-grained process

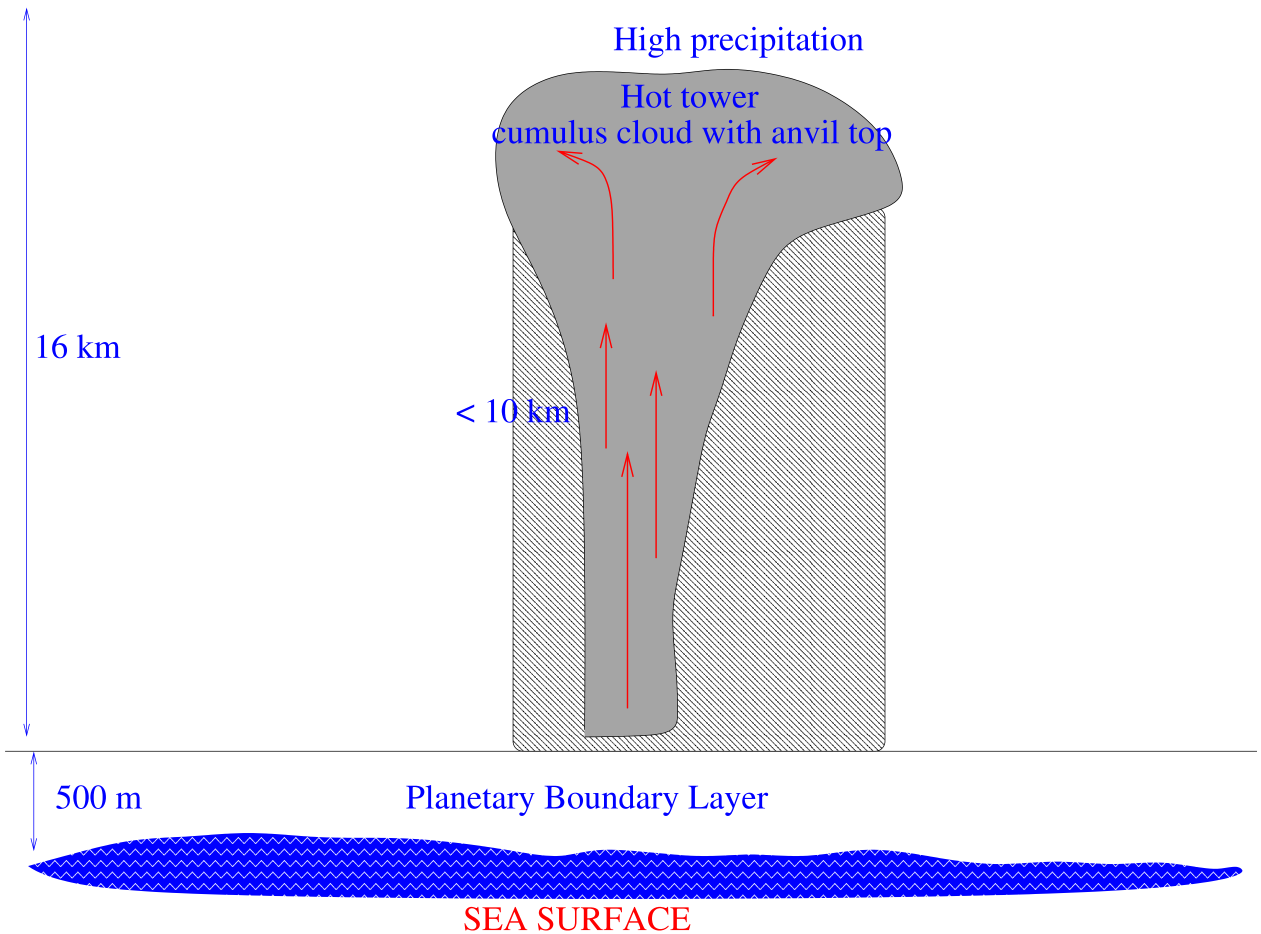
- Canonical invariant Gibbs measure:
  $G_{m,q,\beta}(\eta) = \frac{1}{Z_{m,q,\beta}} e^{\beta \bar{H}(\eta)} P_{m,q}(d\eta)$
- Coarse grained Hamiltonian
  $\bar{H}(\eta) = \frac{U_0}{q-1} \sum_{l \in \Lambda_c} \eta(l)\big(\eta(l) - 1\big) + h \sum_{l \in \Lambda_c} \eta(l)$
- Arrhenius Dynamics lead to birth/death process with Adsorption/Desorption rates:

$$C_a(k,n) = \frac{1}{\tau_I}[q - \eta(k)]$$

$$C_d(k,n) = \frac{1}{\tau_I}\eta(k) e^{-\beta \bar{V}(k)}$$

$$\text{with } \bar{V}(\eta) = \Delta \bar{H}(\eta) = \frac{2U_0}{q-1}\big(\eta(k) - 1\big) + h$$

Stochastic model for CIN coupled into a one-and-half layer model convective parametrization (toy GCM)



The Deterministic Model (Toy GCM): model convective parametrization

- Prognostic Eqns: One vertical baroclinic mode, no rotation

$$\frac{\partial u}{\partial t} - \bar{\bar{\alpha}} \frac{\partial \theta}{\partial x} = -Friction \qquad \frac{\partial \theta}{\partial t} - \bar{\alpha} \frac{\partial u}{\partial x} = Q_c - Q_R$$

$$h \frac{\partial \theta_{eb}}{\partial t} = -D + E \qquad H \frac{\partial \theta_{em}}{\partial t} = D - Q_R$$

Convective heating:

$$Q_c = M \sigma_c \big((\mathrm{CAPE})^+\big)^{1/2}$$

CAPE $\propto \theta_{eb} - \gamma \theta$.

σ_c called *area fraction of deep convection*, plays key role in linear stability.

Coupling Stochastic model into toy GCM

- Order parameter modifies CAPE flux:

$$\sigma_c = (1 - \overline{\sigma}_I)\sigma_c^+; \;\; \sigma_c^+ = .002$$

- External potential depends on large scale dynamics and thermodynamics: (Good guess)

$$h \propto m_-,$$

Downward mass flux $\propto$ Convective mass flux

- convective events build CIN by downdraft cooling of boundary layer and/or convective heating of middle troposphere (stabilization)

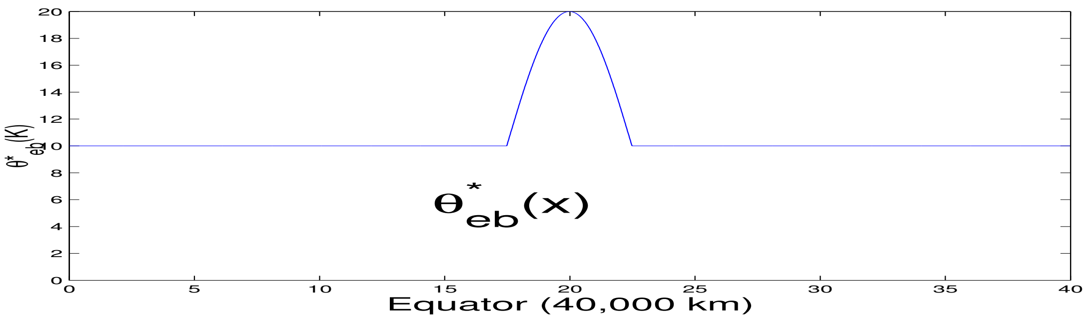
- Other choices of h are possible.

Nonlinear Simulations with Toy GCM

Walker circulation set-up:

mimicking the Indian Ocean/Western Pacific warm pool

$$\frac{\theta^*_{eb}(x)}{\theta^{*,0}_{eb}} = 1 + A_0 \cos\left(\frac{\pi(x-x_0)}{L_0}\right), \quad |x - x_0| < \frac{L_0}{2}$$



- Periodic geometry, $\Delta x = 80$ km
- Initial data: RCE + small random perturbation
- Integrate to statistical equilibrium
- Effects of stochastic model on waves and climate?
- Vary stochastic parameters, βU_0, τ_I, and A_0

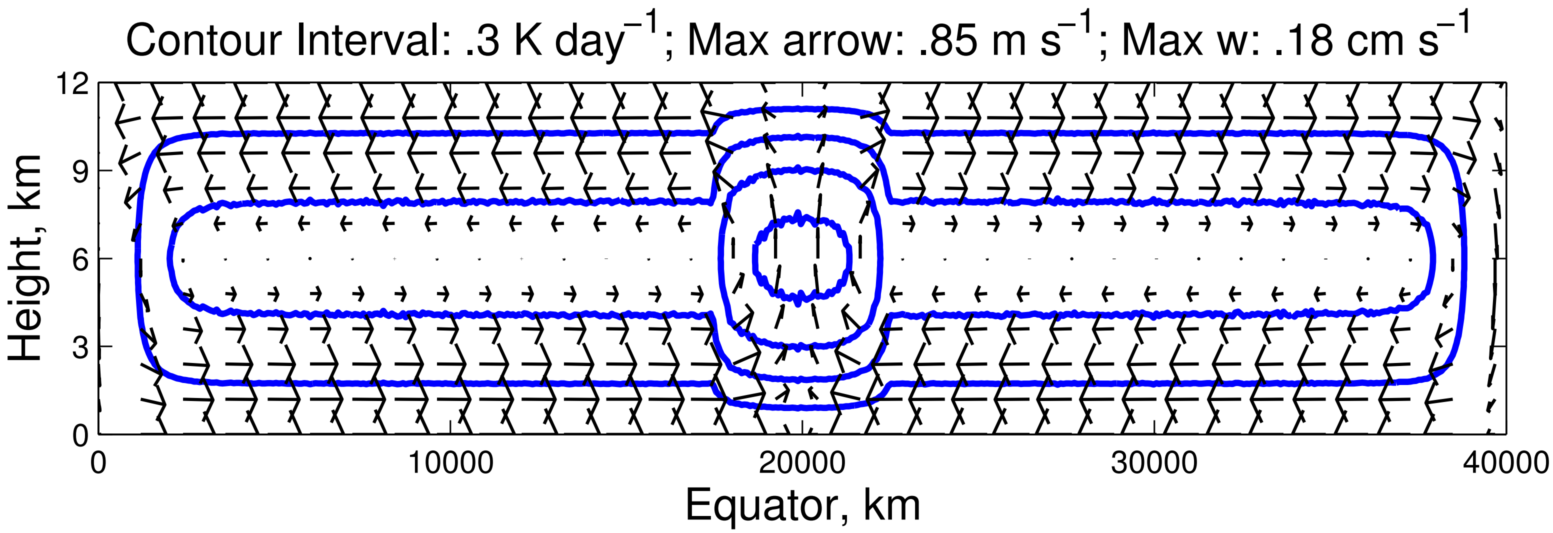
Table 1: Effect of stochastic parameters on climatology and fluctuations, with heating strength $A_0 = .5$.

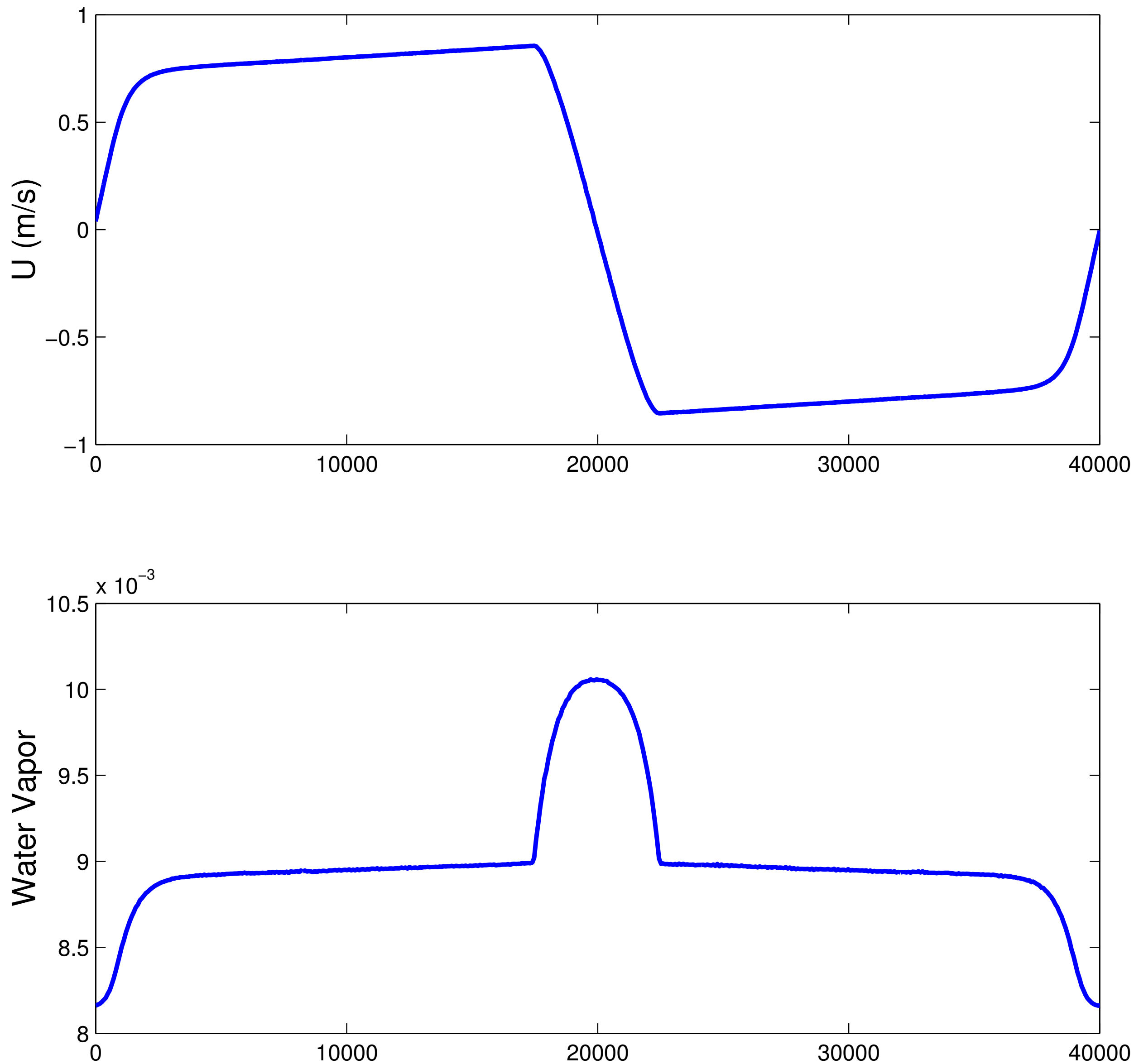
Interac. pot. βU_0	τ_I (days)	$\bar{u}_-$	$\bar{u}_+$	u'_-	u'_+ (m s^{-1})	$\bar{\sigma}_c$	Std. Dev.
1	5	$-.856$	.855	$-.207$	.214	4.55E−04	3.00E−04
1	20	$-.855$	.856	$-.214$	.208	4.55E−04	2.96E−04
.01	5	-1.047	1.046	$-.508$	.486	9.96E−04	3.18E−04
.01	20	-1.048	1.040	$-.804$	.676	9.96E−04	3.15E−04
$-.01$	5	-1.047	1.049	$-.603$	.572	1.00E−03	3.15E−04
$-.01$	10	$-.923$	.920	-4.497	4.429	1.00E−03	3.14E−04
$-.1$	5	$-.816$	.867	-4.820	4.727	1.04E−03	3.11E−04
$-.1$	10	$-.824$	.877	-4.861	4.737	1.04E−03	3.12E−04

Table 2: Same as in Table 1, except for $A_0 = 1$.

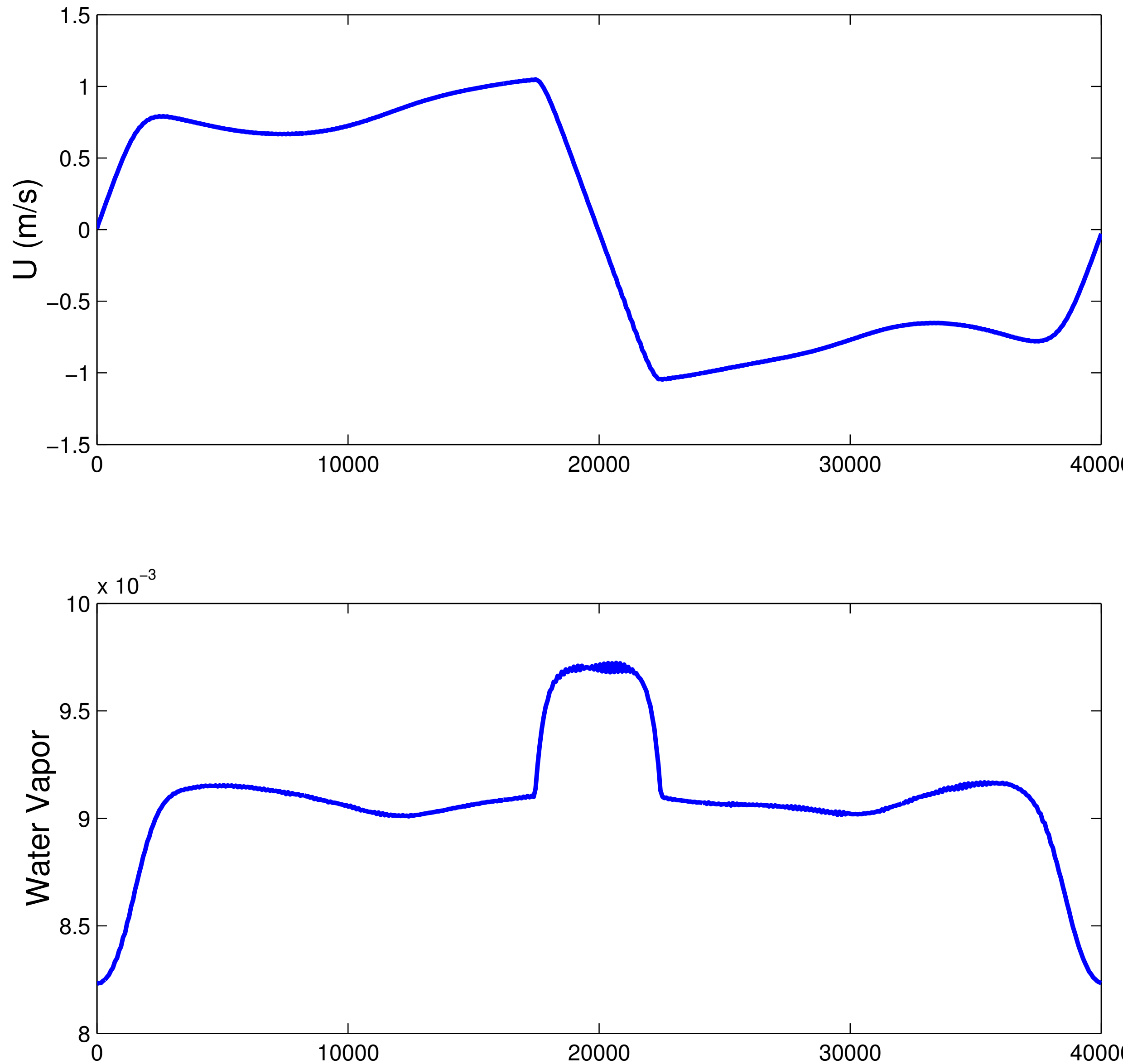
Interac. pot. βU_0	τ_I (days)	$\bar{u}_-$	$\bar{u}_+$	u'_-	u'_+ (m s^{-1})	$\bar{\sigma}_c$	Std. Dev.
1	5	−1.417	1.417	−.536	.436	4.56E−04	3.00E−04
1	20	−1.415	1.417	−.330	.546	4.56E−04	3.00E−04
.01	5	−1.692	1.691	−1.196	1.603	9.96E−04	3.17E−04
.01	20	−1.692	1.691	−1.180	1.266	9.96E−04	3.17E−04
−.01	5	−1.693	1.693	−1.421	1.470	1.00E−03	3.15E−04
−.01	10	−1.693	1.693	−1.277	1.243	1.00E−03	3.16E−04
−.1	5	−1.700	1.699	−.990	1.092	1.04E−03	3.10E−04
−.1	10	−1.700	1.700	−1.447	1.269	1.04E−03	3.07E−04

Typical case: $\beta U_0 = 1, \tau_I = 20$ days, $A_0 = .5$

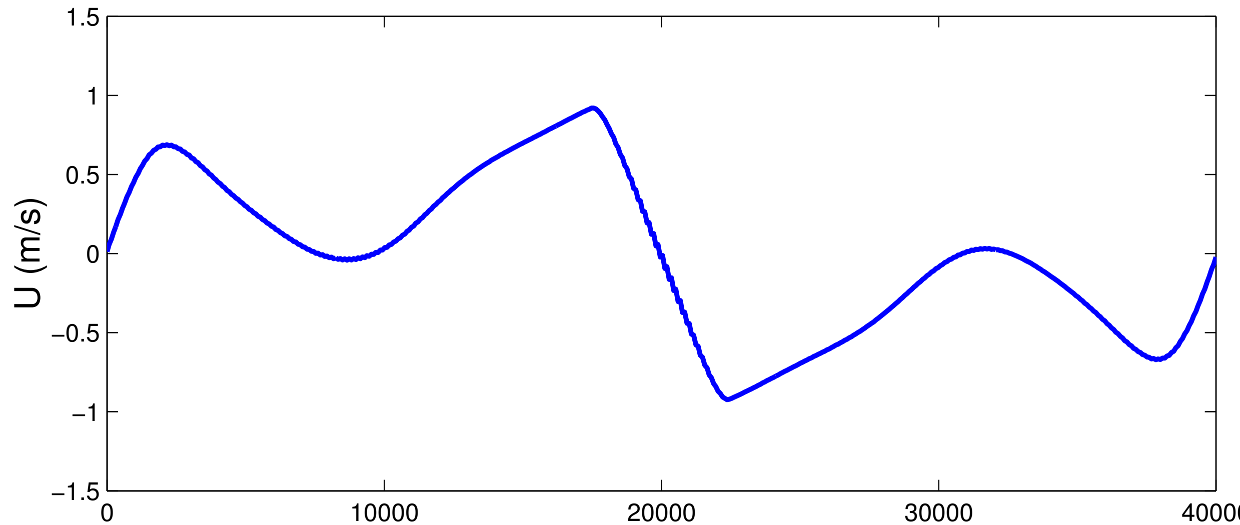


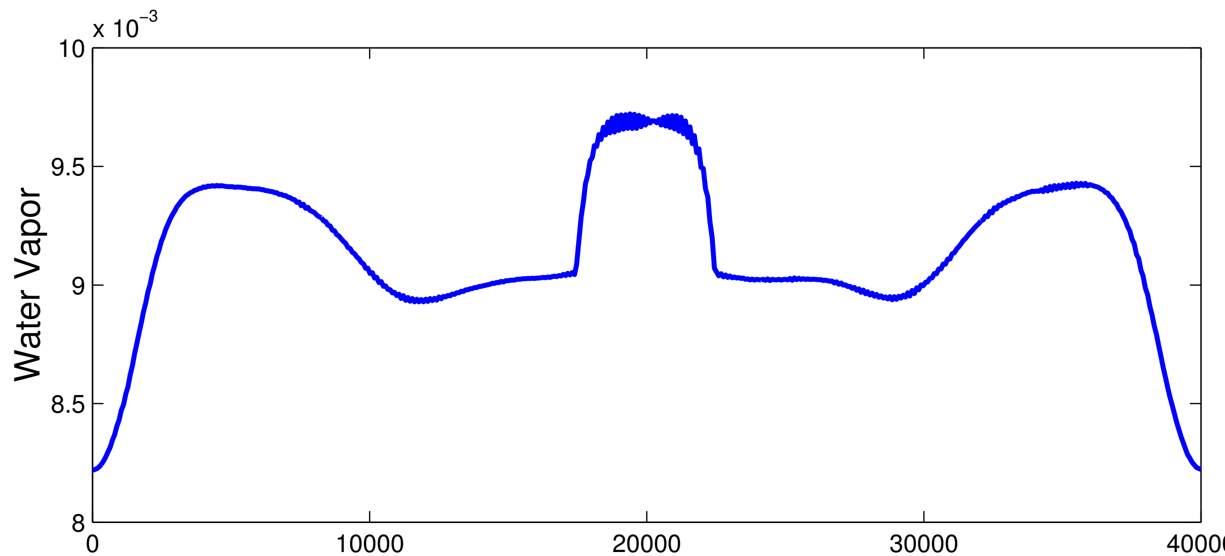

U (m/s)
1
0.5
0
−0.5
−1
0
10000
20000
30000
40000
x 10^{-3}
10.5
10
9.5
9
8.5
8
Water Vapor
0
10000
20000
30000
40000

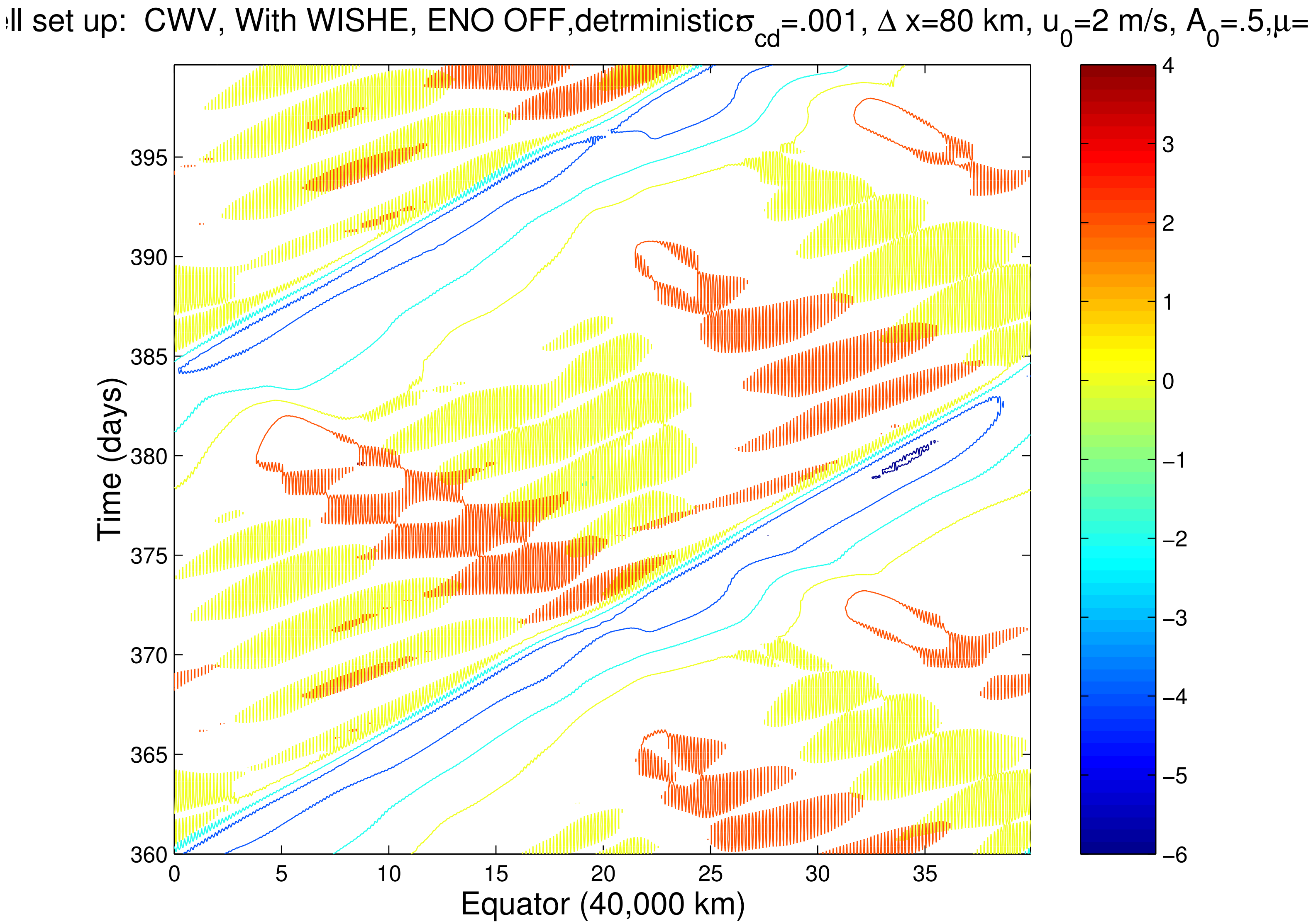
PAC favoring but small CIN time: $\beta U_0 = -.01$, $\tau_I = 5$ days, $A_0 = .5$



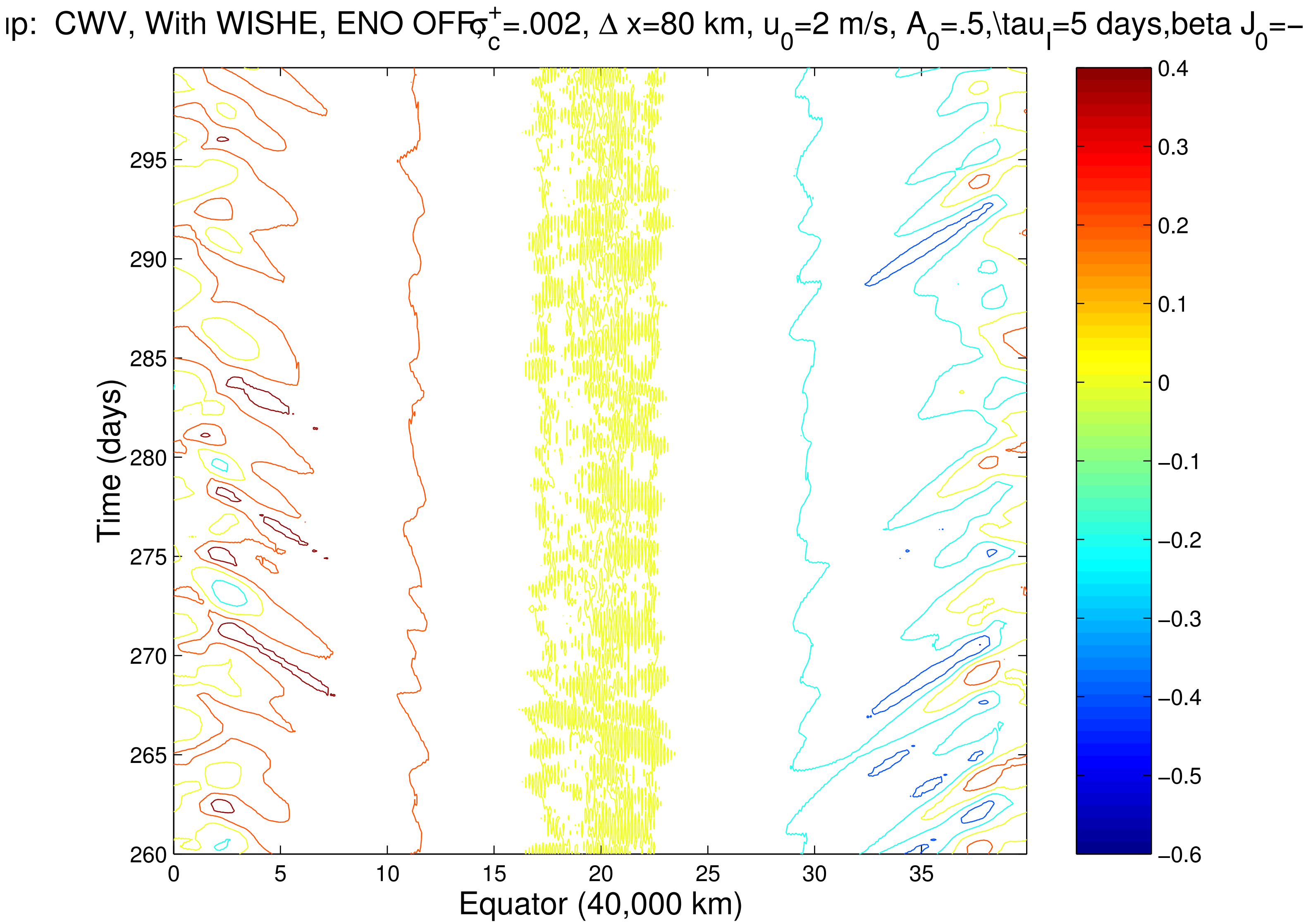
PAC favoring with larger CIN time: $\beta U_0 = -.01$, $\tau_I = 10$ days, $A_0 = .5$



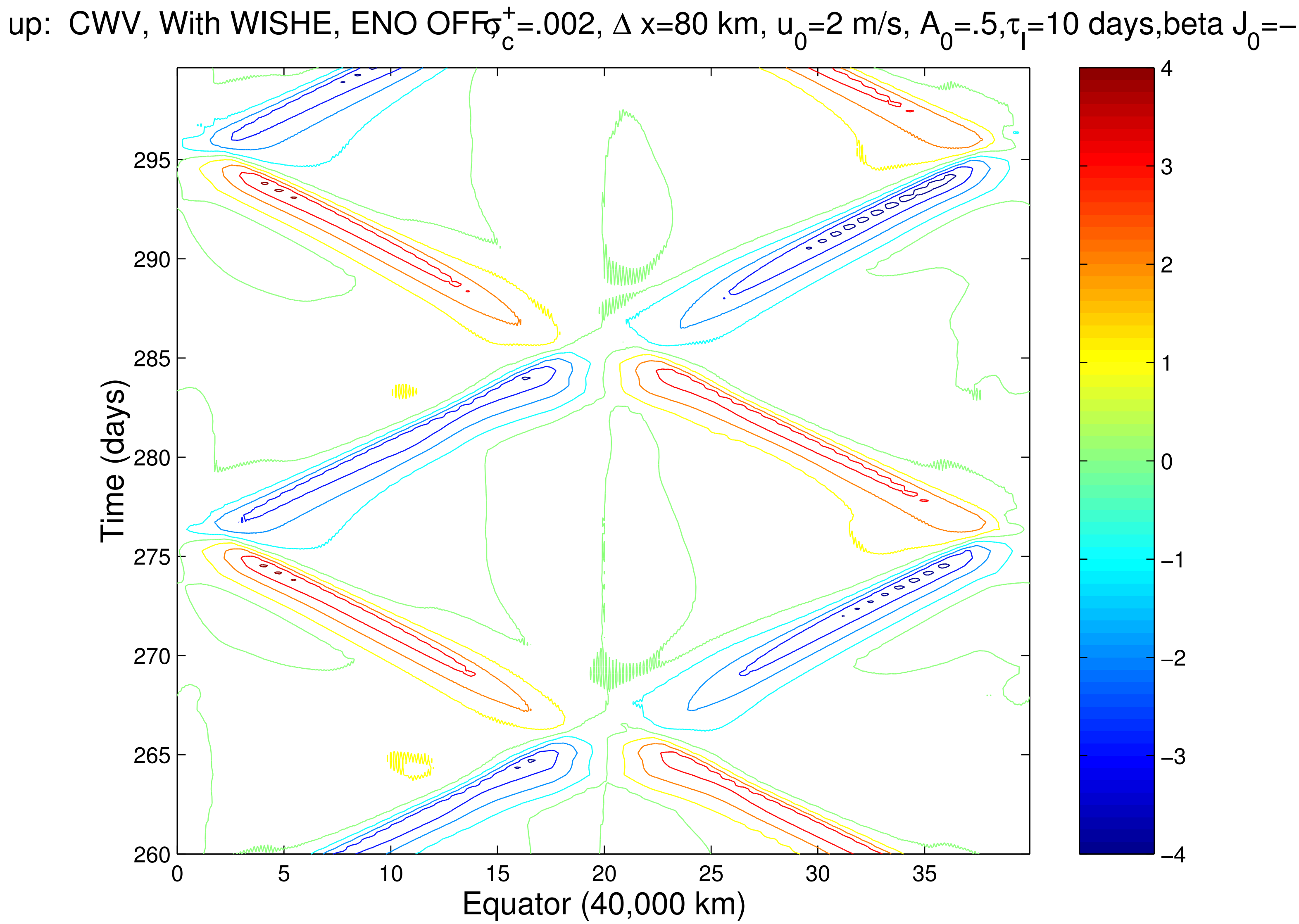




Constant area fraction: $\sigma_c = .001$, $A_0 = .5$



$$\beta U_0 = -.01, \tau_I = 5 \text{ days}, \; A_0 = .5 \; (\bar{\sigma}_c = .001)$$



$$\beta U_0 = -.01, \tau_I = 10 \text{ days}, \ A_0 = .5 \ (\bar{\sigma}_c = .001)$$

- Strong forcing ($A_0 = 1$): Walker cell climatology + weak gravity waves (except for deterministic case)
- Moderate forcing ($A_0 = .5$):
  - Deterministic case: one large scale wave propagating around globe, no Walker cell
  - CIN favoring interaction potential ($\beta U_0 > 0$): Walker cell forms + moderate small scale squall line-like waves
  - As interaction potential decreases strength and length scale of convective waves increases
  - Also sensitive to CIN charac. time (τ_I)
  - PAC favoring int. pot. ($\beta U_0 < 0$): Walker cell destroyed and two symmetric waves propagating far from source
- Stochastic (noise) creates and maintains Walker cell and
- Affects wavelength and strength of waves